

TOPIC 4

The practical computation of Fourier
(e.g., with Matlab)

PART 4

Example 1

Examples with Matlab

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
T=0.2; %%% sampling period  
ws=2*pi/T; %%% sampling frequency/rate/speed  
disp(['T = ', num2str(T)])  
disp(['ws = ', num2str(ws)])  
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
ts=0:T:10;  
W=5; %%% frequency of the signal  
xn=cos(W*ts);
```

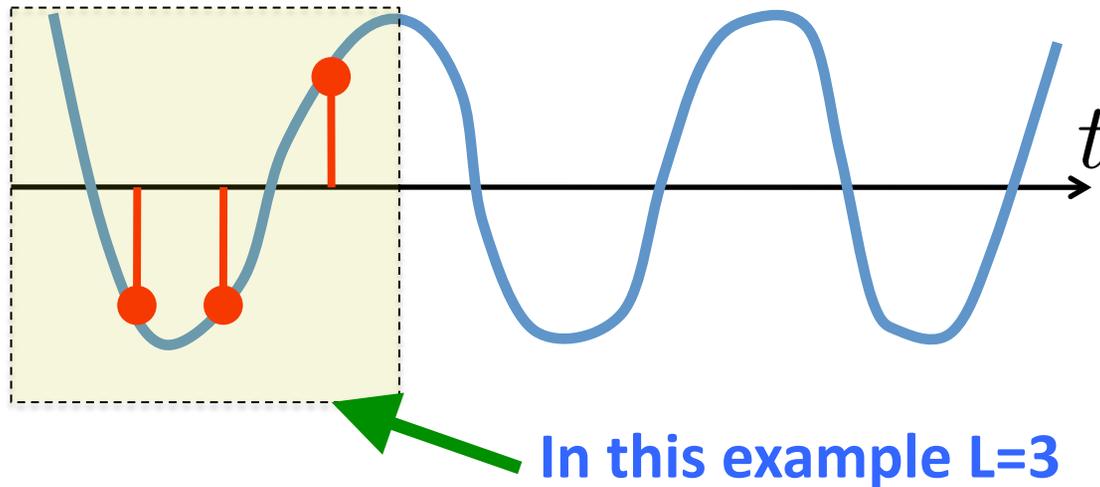
Just write on the command line

Our finite-length sequence of values $x[n]$ (sampled signal, data); we will consider real signals

RECALL

$$x(t) = \cos(Wt)$$

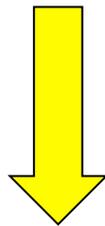
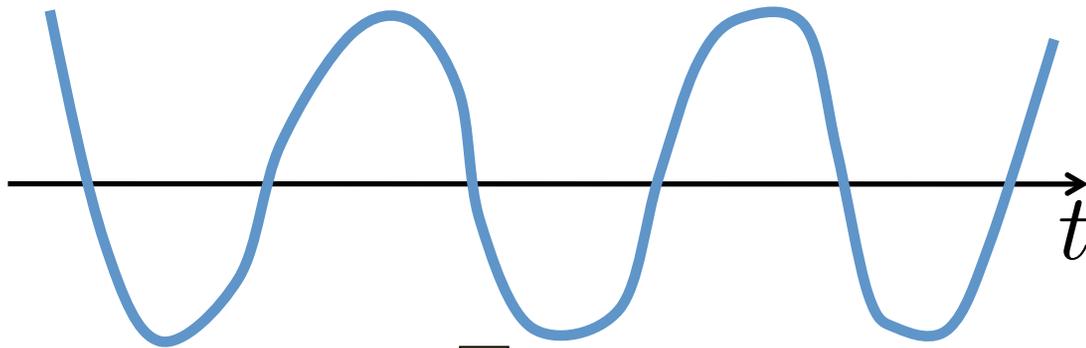
We see “samples” in a temporal
“window”... (a piece)



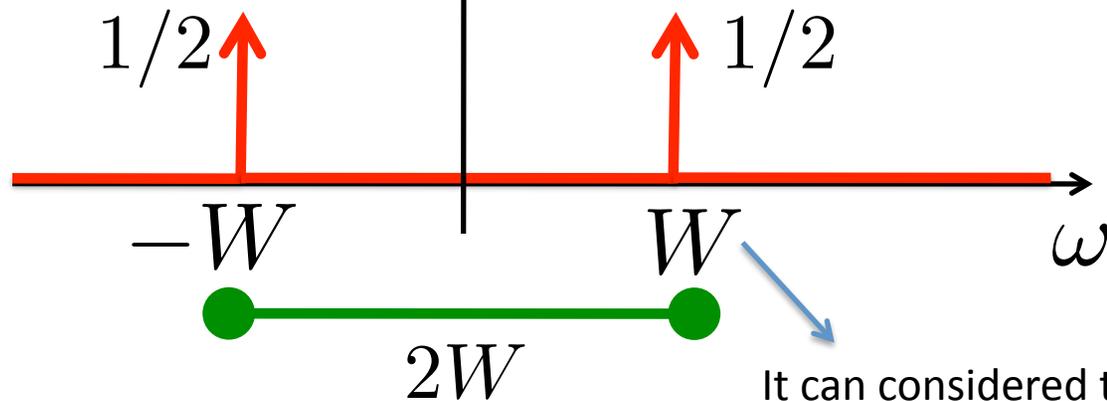
$$x[n] = x(nT) = \cos(WnT)$$

RECALL

$$x(t) = \cos(Wt)$$

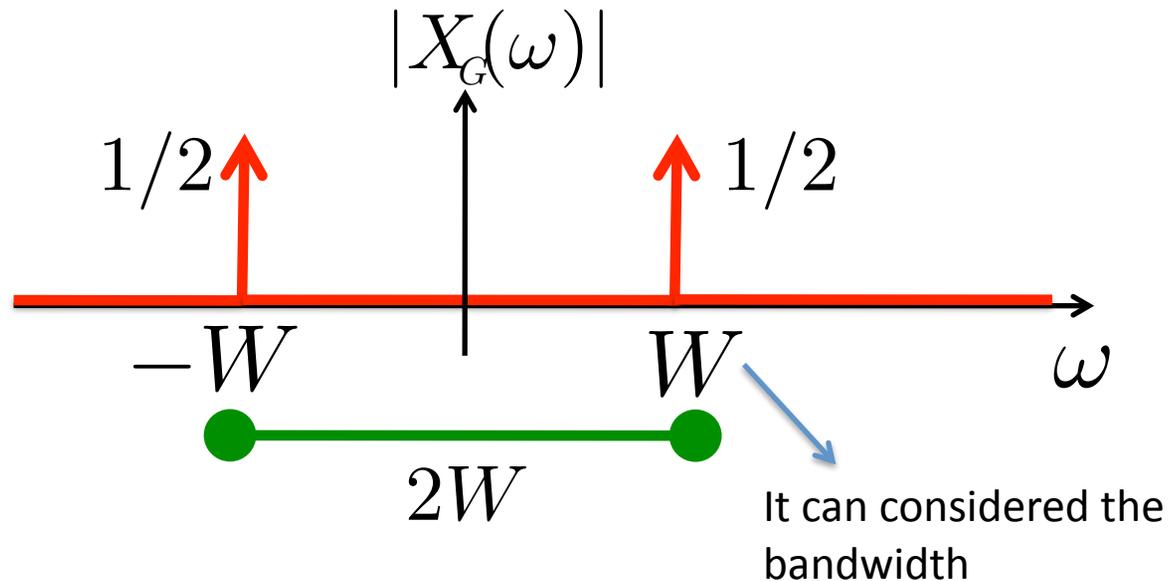


$$|X_G(\omega)|$$



It can considered the bandwidth

RECALL: do we satisfy Nyquist?



$$T = 0.2$$

$$2W = 10$$

$$\omega_s = \frac{2\pi}{T} = 31.4159$$

$$\omega_s \geq 2W$$

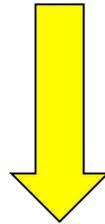
YES, we fulfill Nyquist !

RECALL

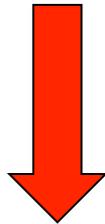
However, we have/see just a FINITE number of values... sampled at each time T :

Just a finite number of values

$$x(t) = \cos(Wt)$$



$$x[n] = x(nT) = \cos(WnT)$$



$$X(\Omega)$$

But we compute DFT/FFT:



$$X_N[k]$$

RECALL

Matlab returns $N \geq L$ complex numbers:

$$X_N[0], X_N[1], X_N[2], \dots, X_N[N-1]$$

$$\downarrow$$
$$0$$

$$\downarrow$$
$$\Omega_0$$

$$\downarrow$$
$$2\Omega_0$$

$$\downarrow$$
$$(N-1)\Omega_0$$



$$\Omega_0 = \frac{2\pi}{N}$$

$$N \geq L$$

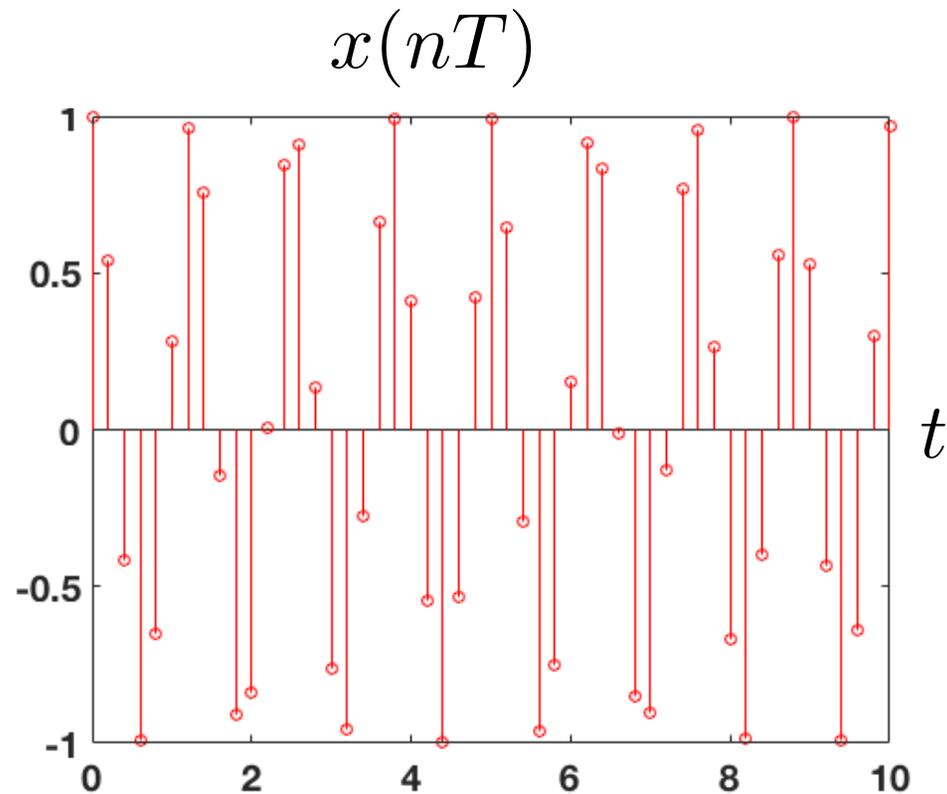
In this
example
 $L=51$

$$X_N[k] = X(\Omega) \Big|_{k\Omega_0} = X(k\Omega_0)$$

$$X_N[k] = X(\Omega) \Big|_{k \frac{2\pi}{N}} = X\left(k \frac{2\pi}{N}\right)$$

Examples with Matlab

```
figure  
stem(ts,real(xn),'r')  
title('DATA x[n]')  
set(gca,'fontSize',20,'fontWeight','Bold')
```



Examples with Matlab

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
L=length(xn); %%% length of the signal  
N=L; %%% you can choose N>=L  
FXs=fft(xn,N);  
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

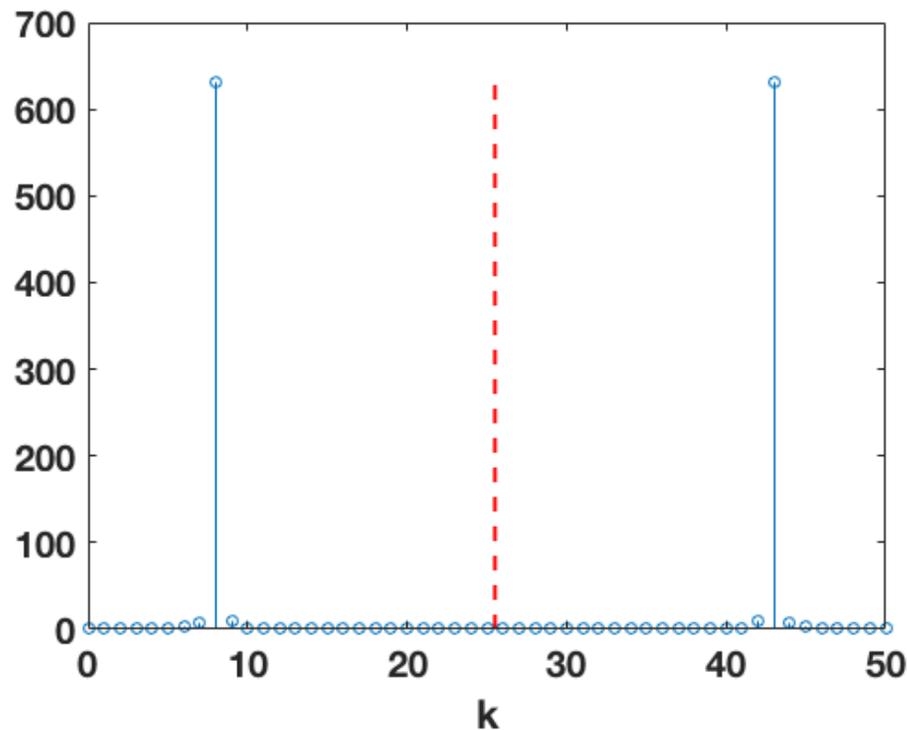
Fast Fourier Transform (FFT) $X_N[k]$

In this example: L=51

Examples with Matlab

```
figure
stem(0:N-1,abs(FXs).^2)
hold on
plot([N/2 N/2],[0 max(abs(FXs).^2)],'r--','LineWidth',2)
set(gca,'fontSize',20,'fontWeight','Bold')
xlabel('k')
xlim([0 N-1])
```

$$|X_N[k]|$$



L=51

N=L

In the “Omega domain” (discrete time)

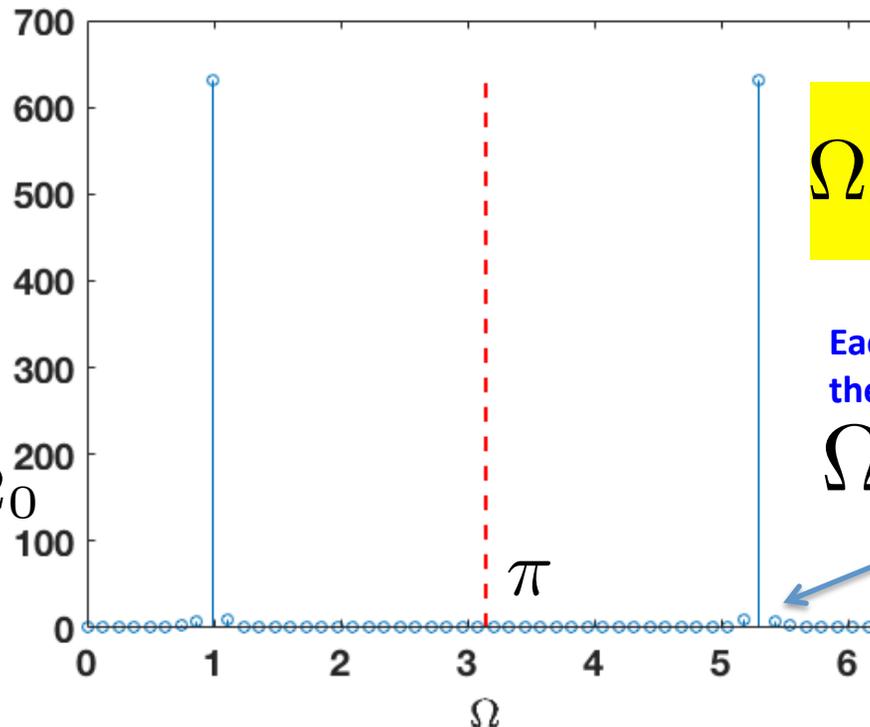
```
figure
Omega=(0:N-1)*(2*pi/N);
stem(Omega,abs(FXs).^2)
hold on
plot([pi pi],[0 max(abs(FXs).^2)],'r--','LineWidth',2)
set(gca,'fontSize',20,'fontWeight','Bold')
xlabel('\Omega')
xlim([0 2*pi])
```

N=L=51

$$|X(k\Omega_0)|$$



$$|X(\Omega)|_{\Omega=k\Omega_0}$$



$$\Omega_0 = \frac{2\pi}{N} = 0.1232$$

Each point corresponds to the frequency:

$$\Omega_k = k\Omega_0$$

2π

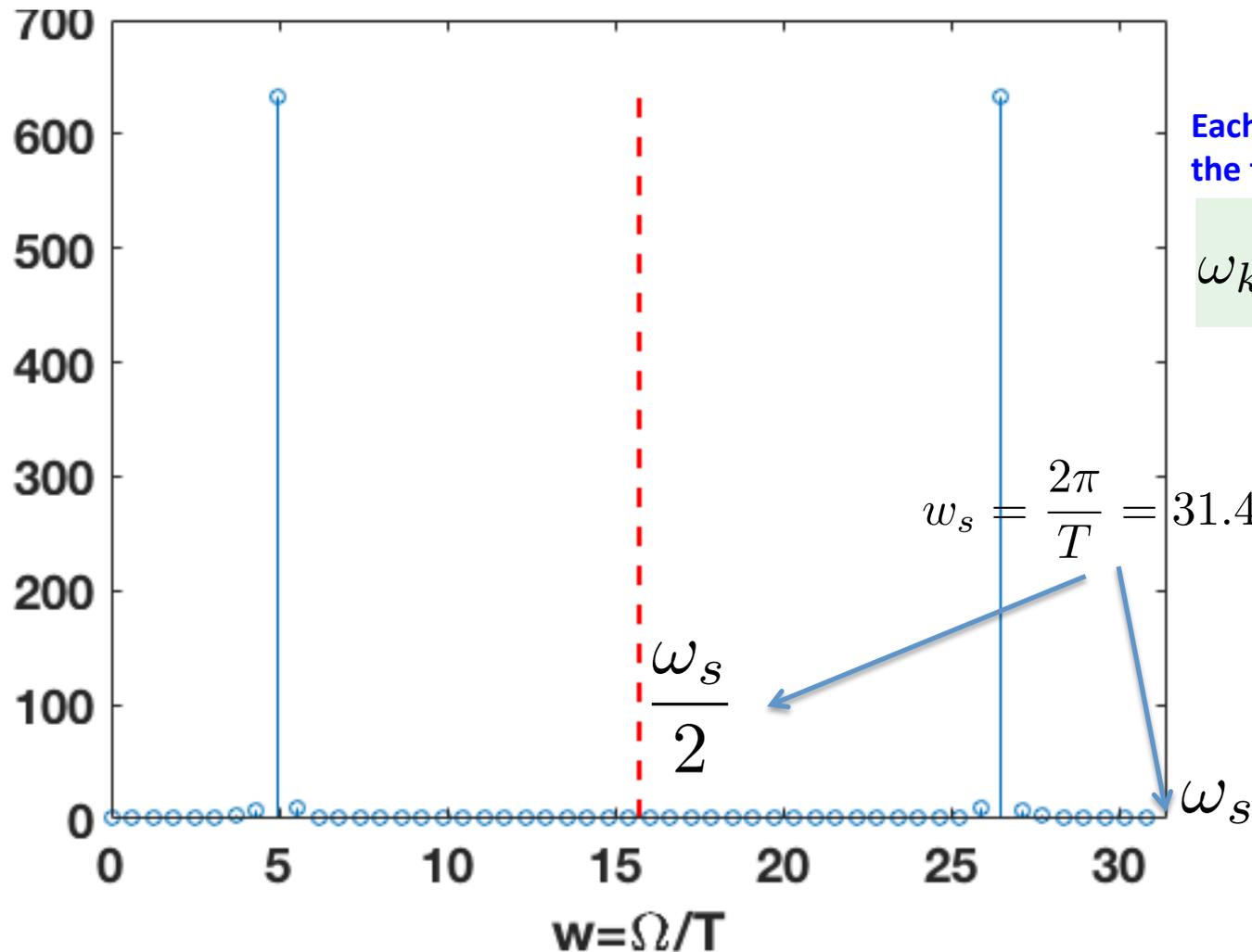
In the “omega domain” (continuous time)

```
figure
w=0:(ws/2)/(N/2):ws-(ws/2)/(N/2);
stem(w,abs(FXs).^2)
hold on
plot([pi/T pi/T],[0 max(abs(FXs).^2)],'r--','LineWidth',2)
set(gca,'fontSize',20,'fontWeight','Bold')
xlabel('w=\Omega/T')
xlim([0 2*pi/T])
title(['T = ', num2str(T)])
```

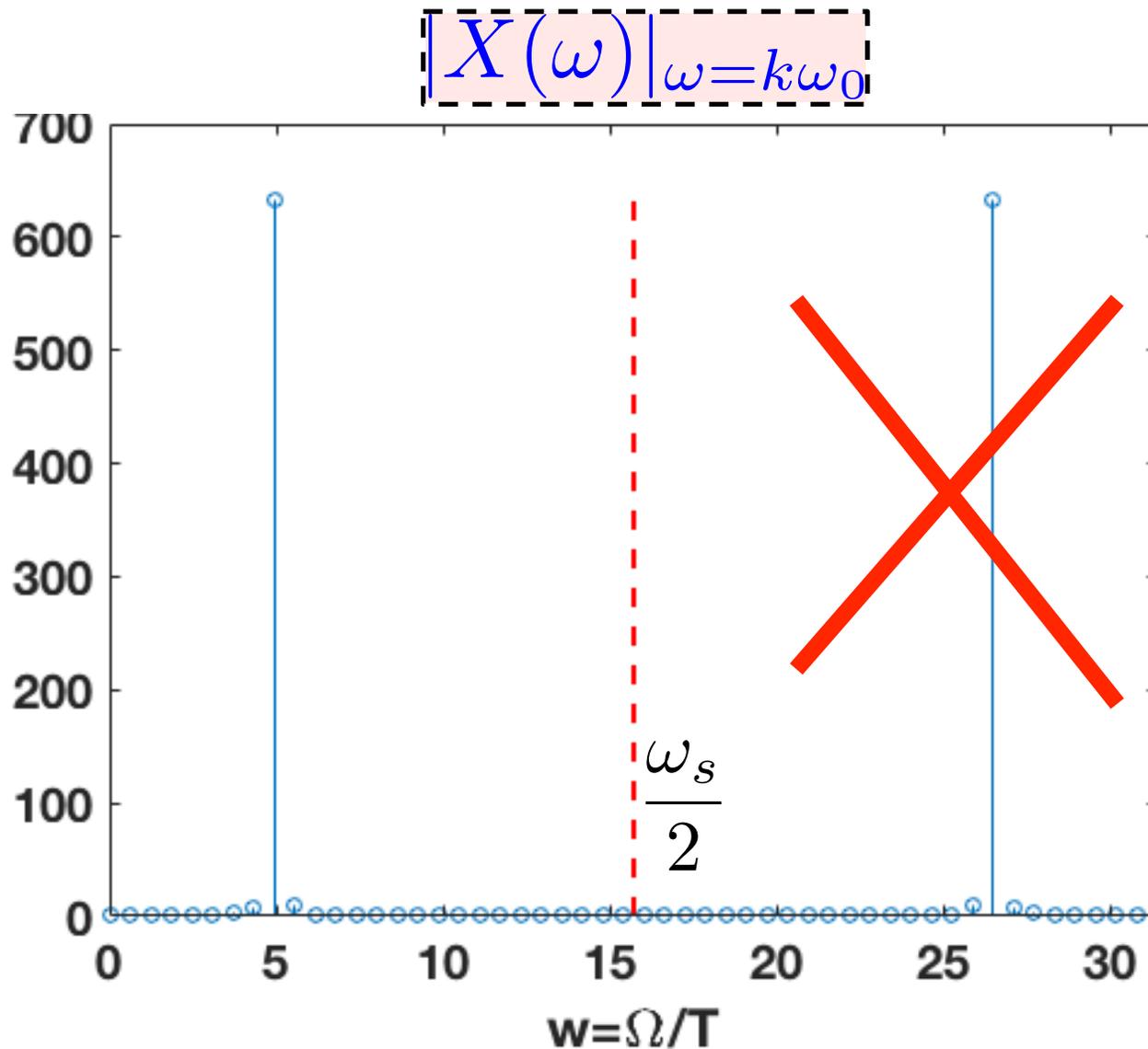
In the “omega domain” (continuous time)

$$|X(\omega)|_{\omega=k\omega_0}$$

$$\omega_0 = \frac{\Omega_0}{T} = \frac{2\pi}{NT} = 0.6160$$



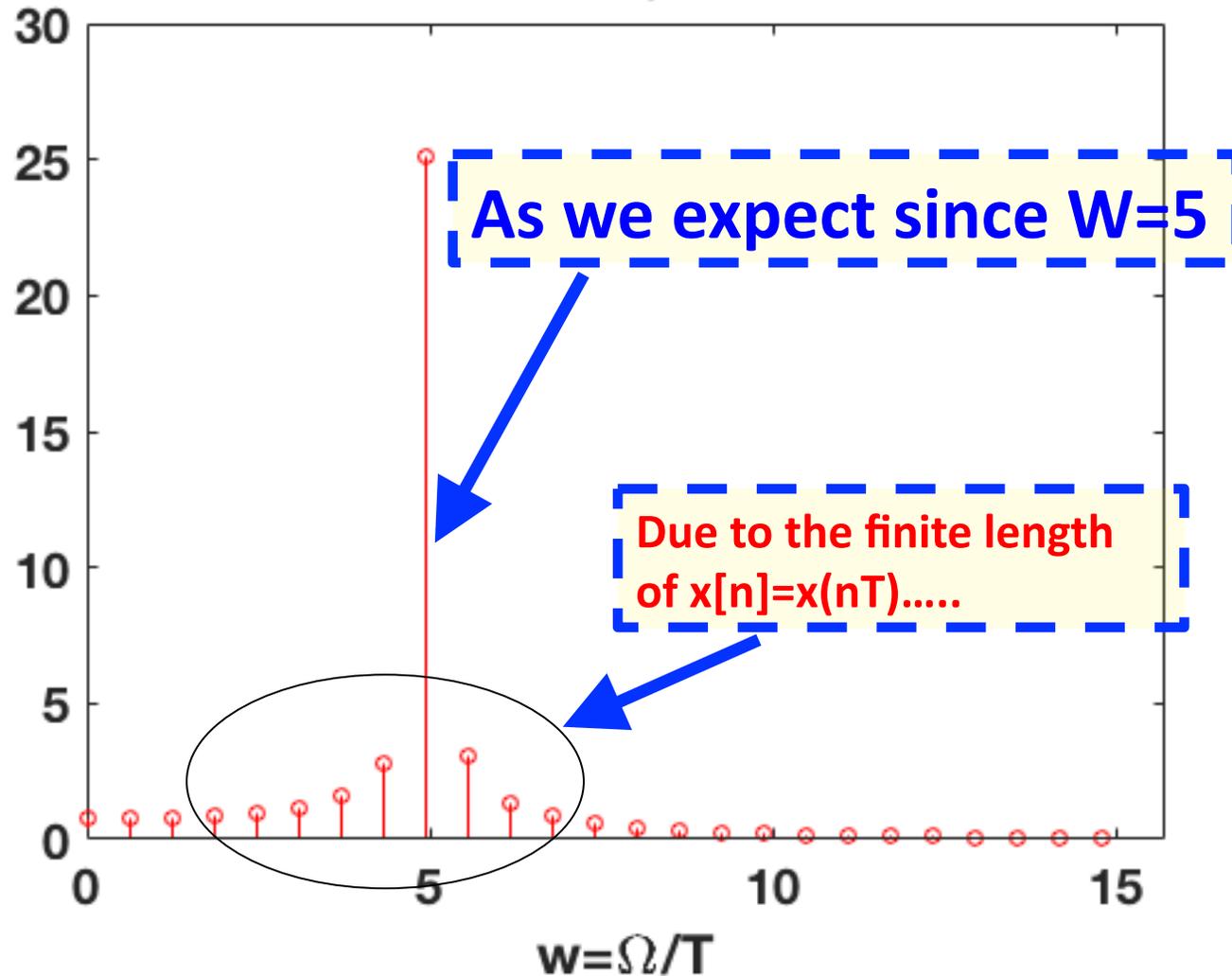
In the “omega domain” (continuous time)



In the “omega domain” (continuous time)

$$X(\omega) \Big|_{\omega = k\omega_0}$$

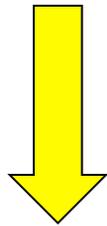
$T = 0.2$



Example 2

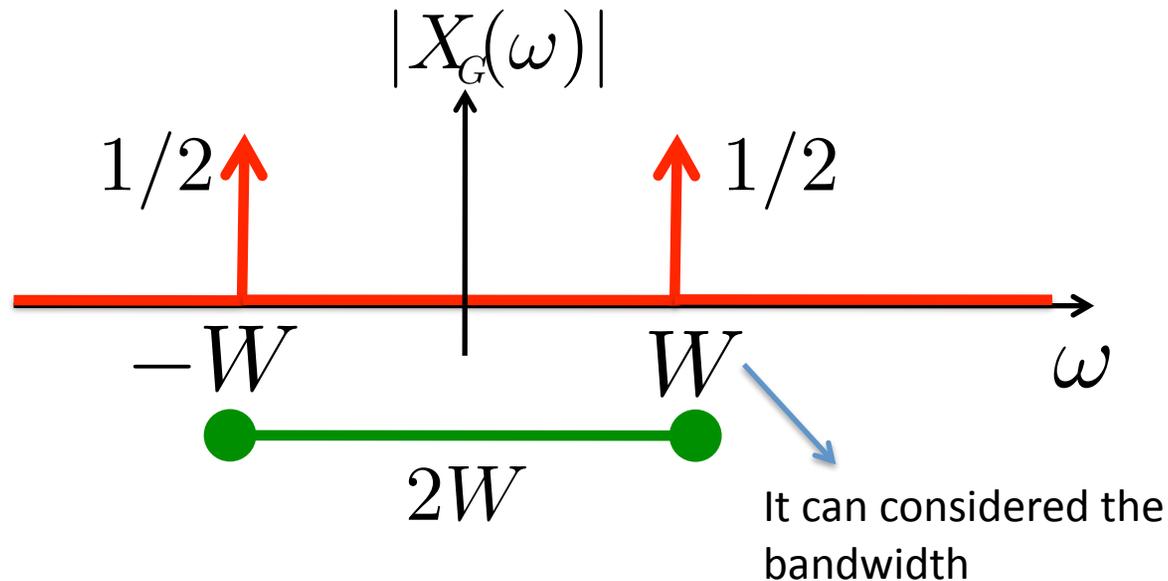
Now consider....

$$x(t) = \cos(Wt)$$



$$W = 20$$

NOW: do we satisfy Nyquist?



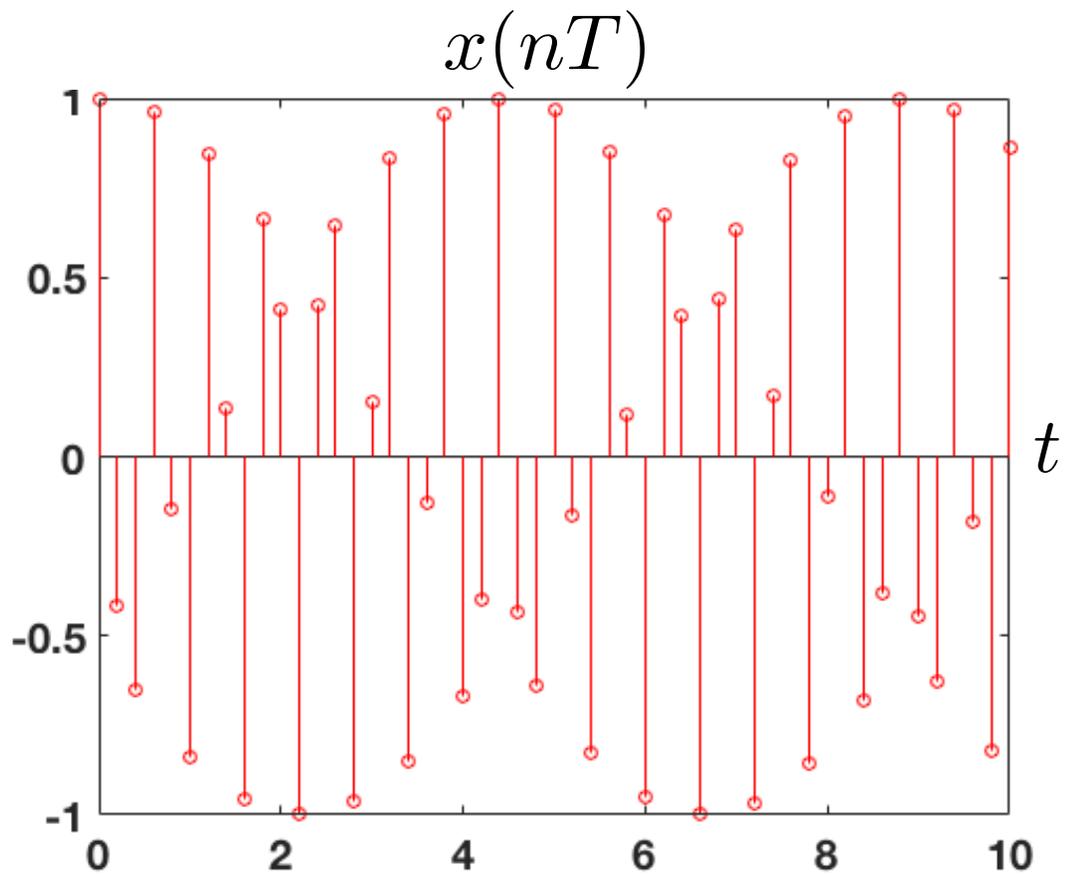
$$T = 0.2 \quad 2W = 40$$

$$\omega_s = \frac{2\pi}{T} = 31.4159$$

$$\omega_s \geq 2W$$

NO, we DO NOT fulfill Nyquist !

Examples with Matlab



Examples with Matlab

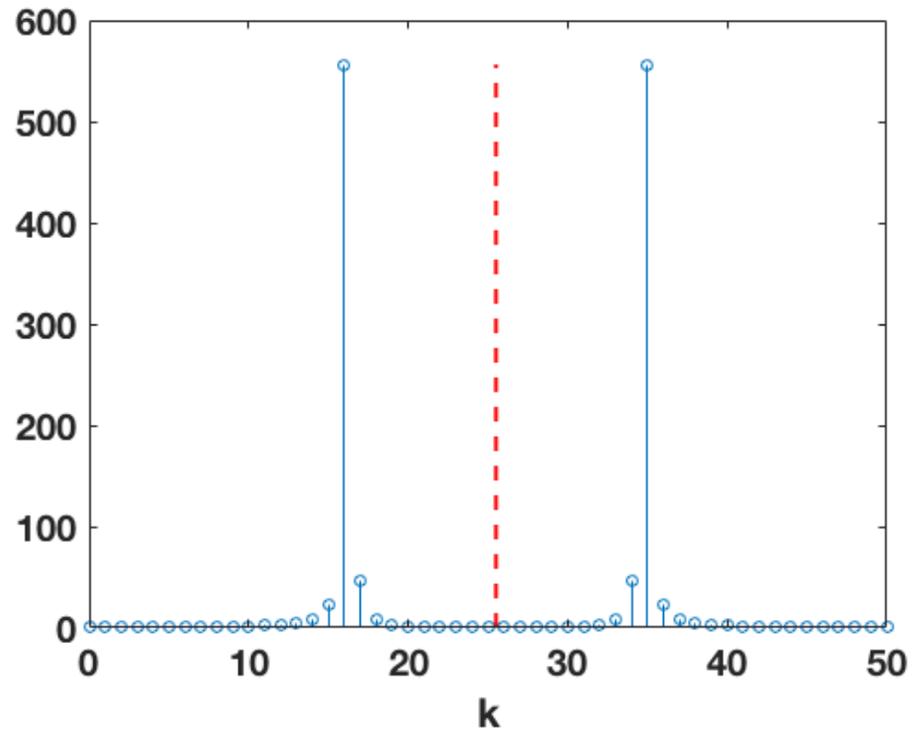
```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
L=length(xn); %%% length of the signal  
N=L; %%% you can choose N>=L  
FXs=fft(xn,N);  
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Fast Fourier Transform (FFT) $X_N[k]$

AGAIN: L=51

Examples with Matlab

$$|X_N[k]|$$



L=51

N=L

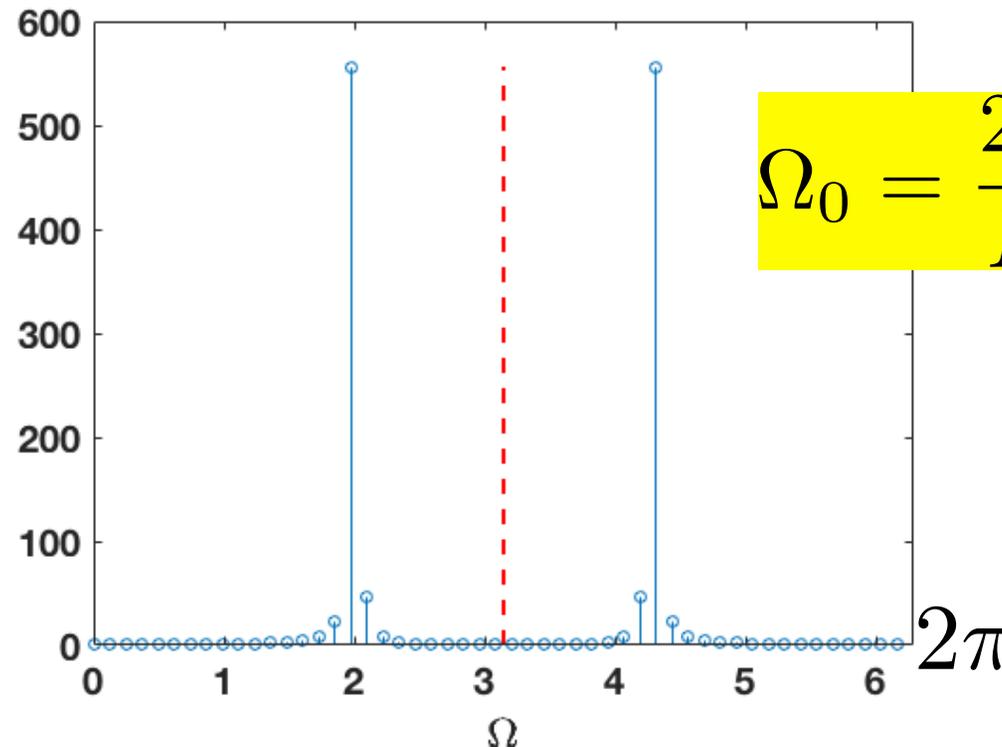
In the “Omega domain” (discrete time)

N=L=51

$$|X(k\Omega_0)|$$

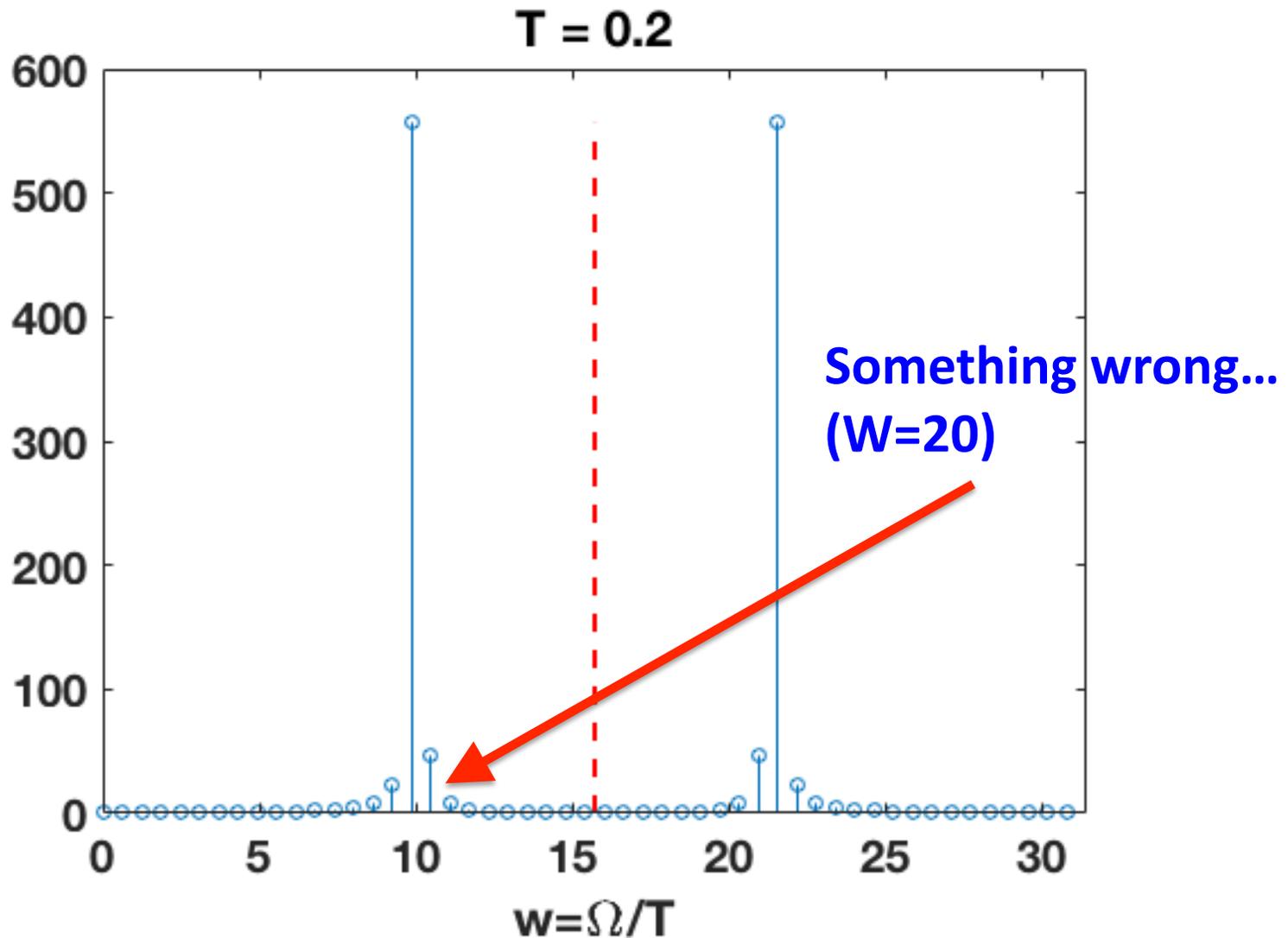


$$|X(\Omega)|_{\Omega=k\Omega_0}$$



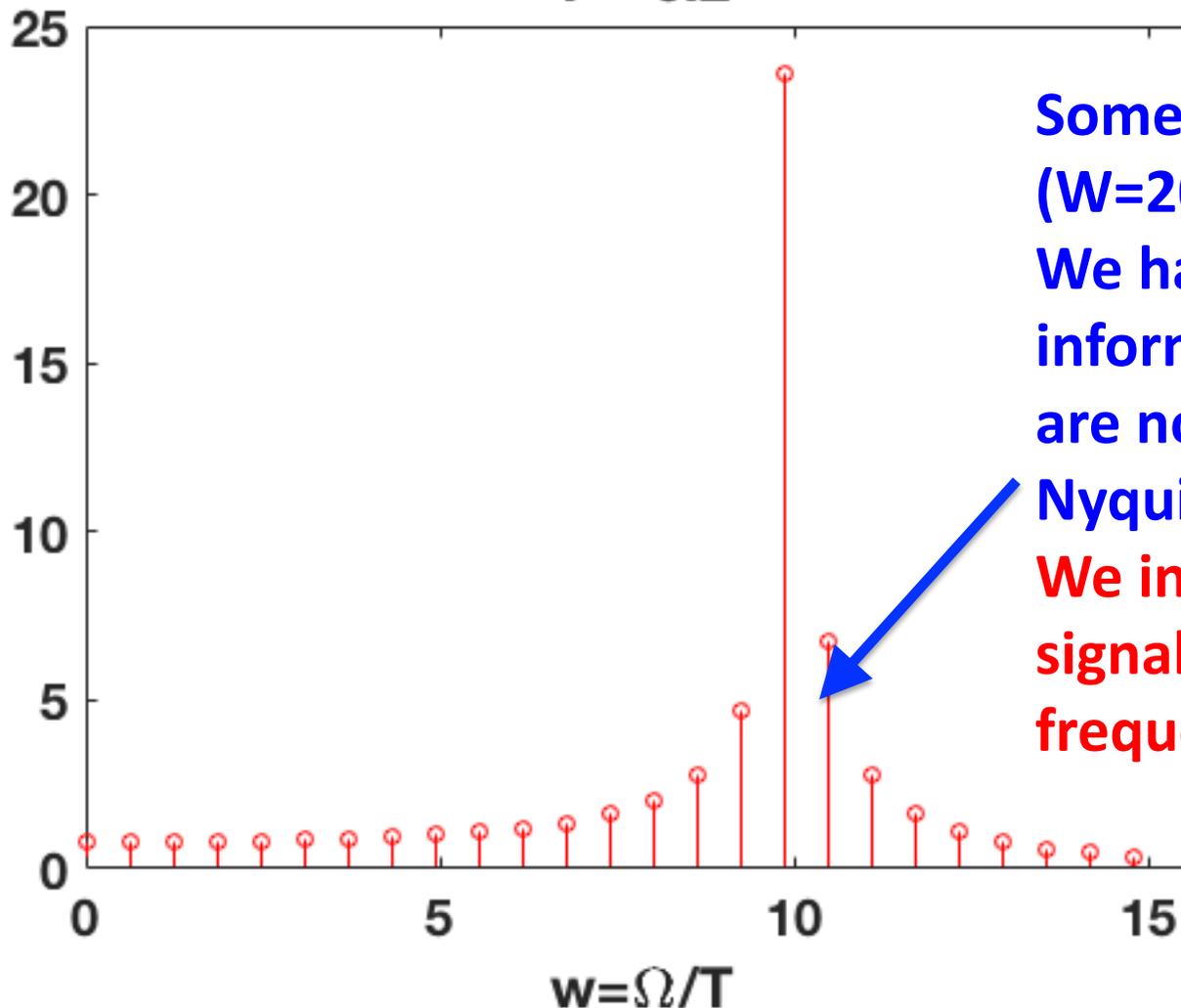
$$\Omega_0 = \frac{2\pi}{N} = 0.1232$$

In the “omega domain” (continuous time)



WE ARE NOT FULFILLING NYQUIST !!!!

$T = 0.2$



Something wrong...

($W=20$)

We have lost
information, since we
are not satisfying
Nyquist...

We interpret $x(t)$ as a
signal with lower
frequency

**Play with the code...
that I gave you...**