Solved Problems - Sampling (discrete time)

Linear systems and circuit applications

Discrete Time Systems

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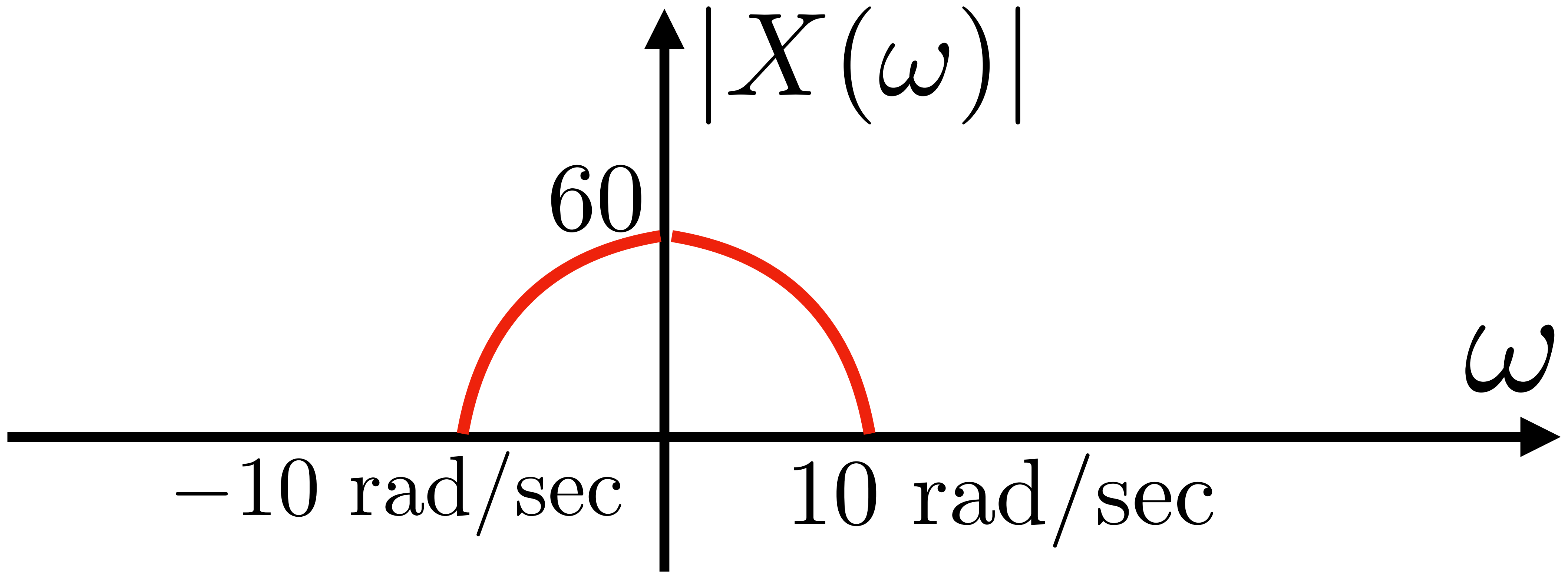
Example 1

Let consider the following module of FT of a signal x(t)



Is it a bandlimited signal ? explain your answer.

Find the minimum sampling *rate* for the signal which allows a perfect reconstruction (in interpolation).

Example 1

It is a bandlimited signal since the module of the FT is non-zero only in a finite range of values of frequencies. Indeed, we have

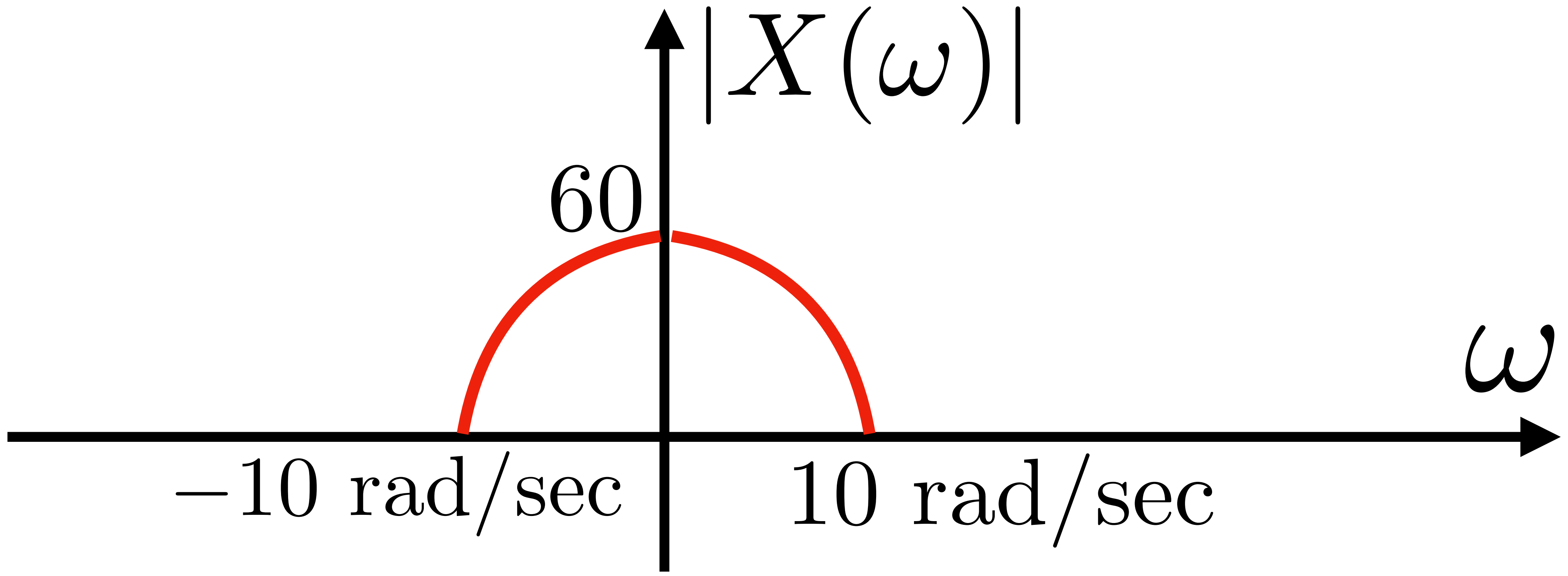
$$|X(\omega)| > 0 \qquad \text{only in } |\omega| < 10 \text{ rad/sec}$$

For the Nyquist theorem, the minimum sampling rate is:

$$W = 10 \text{ rad/sec}$$
$$\omega_s \geq \text{Rate-min} = 2W = 20 \text{ rad/sec}$$

Example 2

Let consider the following module of FT of a signal x(t)



Find the minimum sampling *rate/frequency* for the signal which allows a perfect reconstruction (in interpolation), expressing the rate in Hertz (Hz).

Example 2

For the Nyquist theorem, the minimum sampling rate is:

$$W = 10 \text{ rad/sec}$$

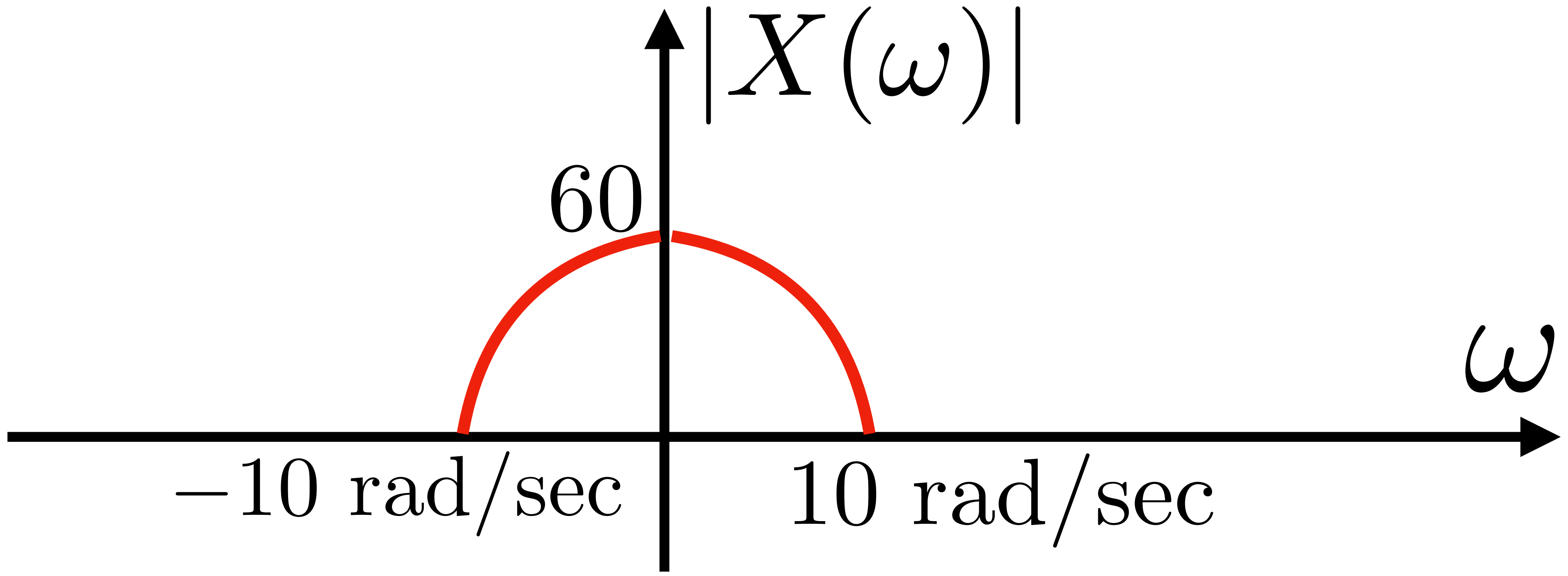
$$\omega_s \geq \text{Rate-min} = 2W = 20 \text{ rad/sec}$$

In Hertz (Hz), we have:

$$f_s = \frac{\omega_s}{2\pi} = \frac{20}{2\pi} = 3.1831 \quad \text{Hz}$$

Example 3

Let consider the following module of FT of a signal x(t)



What do you can say regarding some features of the signal x(t)?

Find the maximum sampling *period* T for the signal which allows a perfect reconstruction (in interpolation).

Example 3

For sure, we can say:

- the signal is real since the module of the FT is even. We cannot say that the signal is even since we should check that all the FT is even, but we have only the information of the module (and nothing regarding the phase).

- The signal is a smooth signal (no quick variations/oscillations) since has energy in low frequencies (maximum at zero) and its energy is zero for high frequencies.

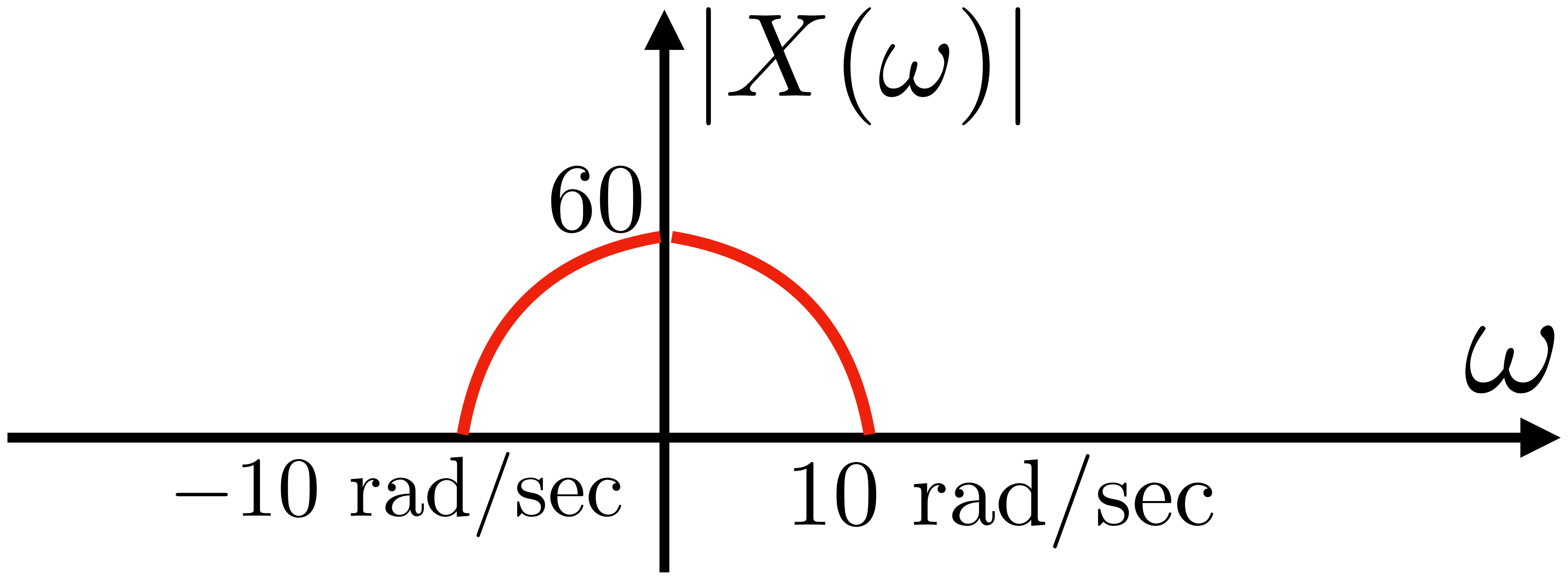
For the Nyquist theorem, the maximum sampling period is:

$$W = 10 \text{ rad/sec}$$

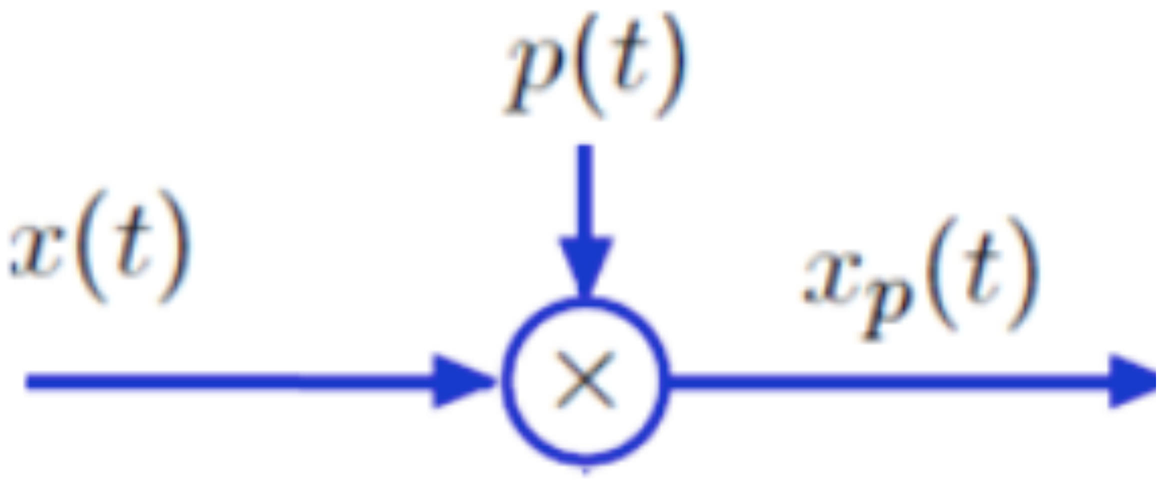
$$T \geq \text{Period-max} = \frac{\pi}{W} = 0.3142 \text{ sec}$$

Example 4

Let consider the following module of FT of a signal x(t)



Show the module of the FT of the continuous signal x_p(t) obtained sampling the signal x(t) (multiplying for a train delta),



$$\boxed{x_p(t) = x(t)p(t)}$$

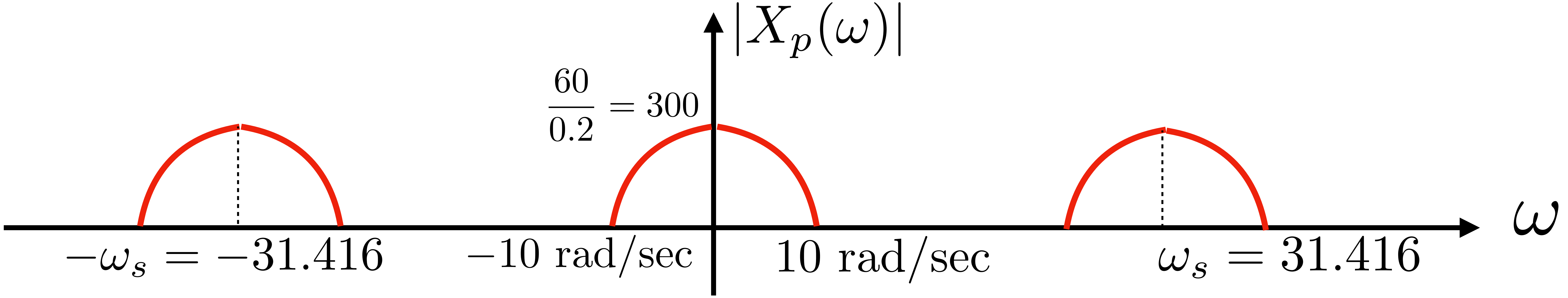
$$\boxed{p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)}$$

with T=0.2 sec.

Example 4

(in the exam, we have to explain every step very well… with comments and explanations…)

It appears “replicas” each $\omega_s = \dfrac{2\pi}{T} = 31.416$ rad/sec :

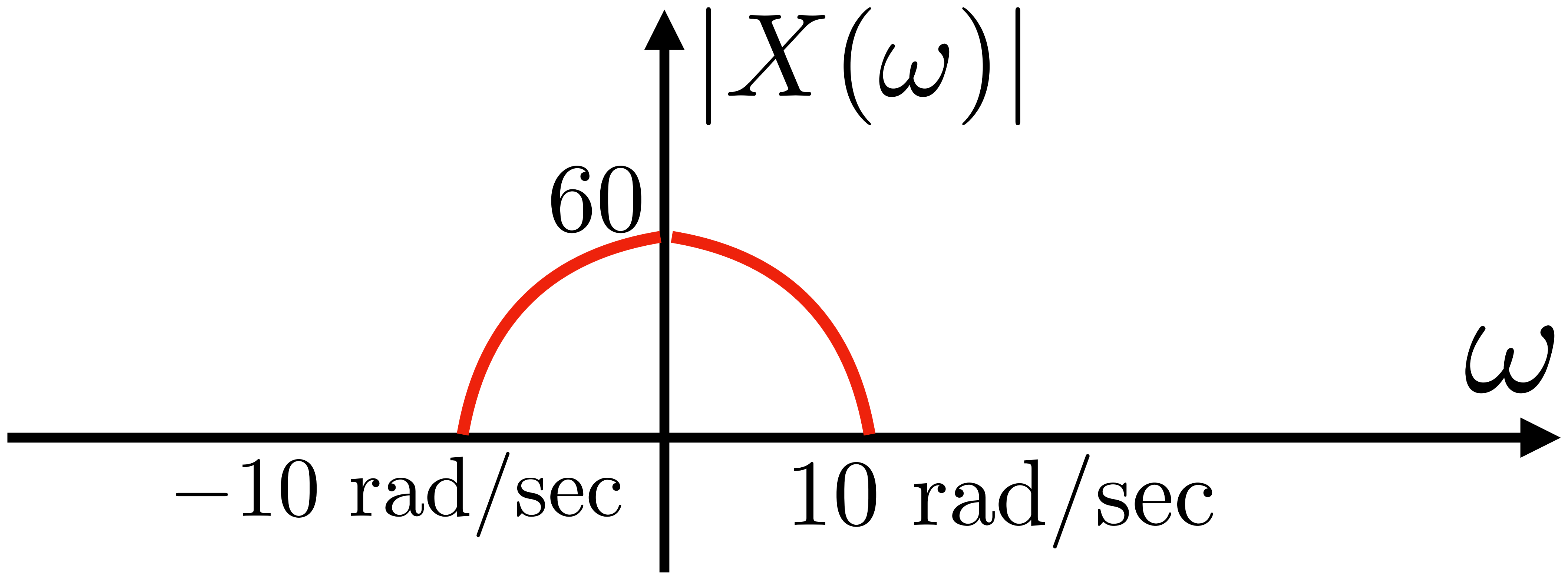


Note that these are just the first replicas (there are an infinite number of replicas)

there is no “aliasing” in this case !!

Example 5

Let consider the following module of FT of a signal x(t)



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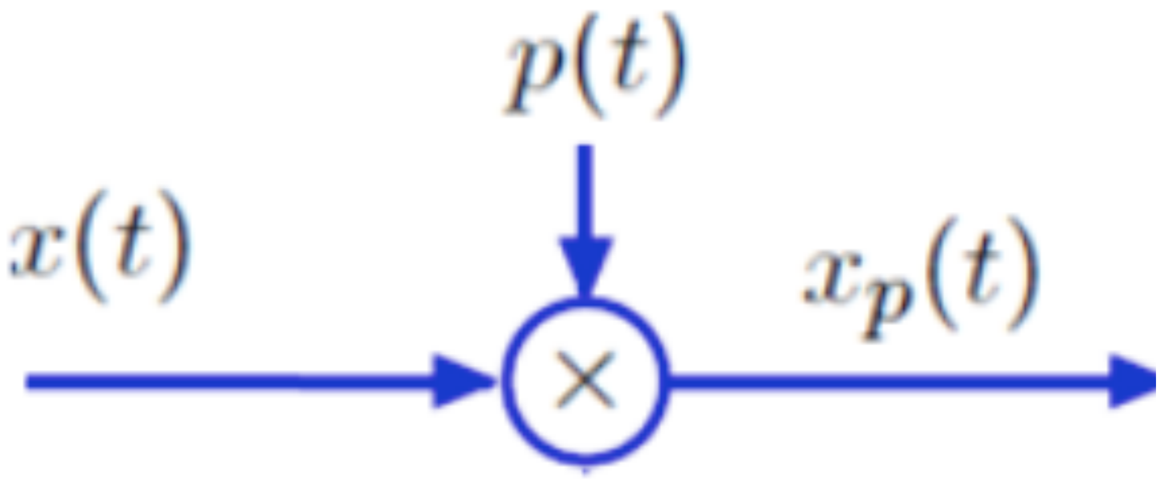


$$x_p(t) = x(t)p(t)$$

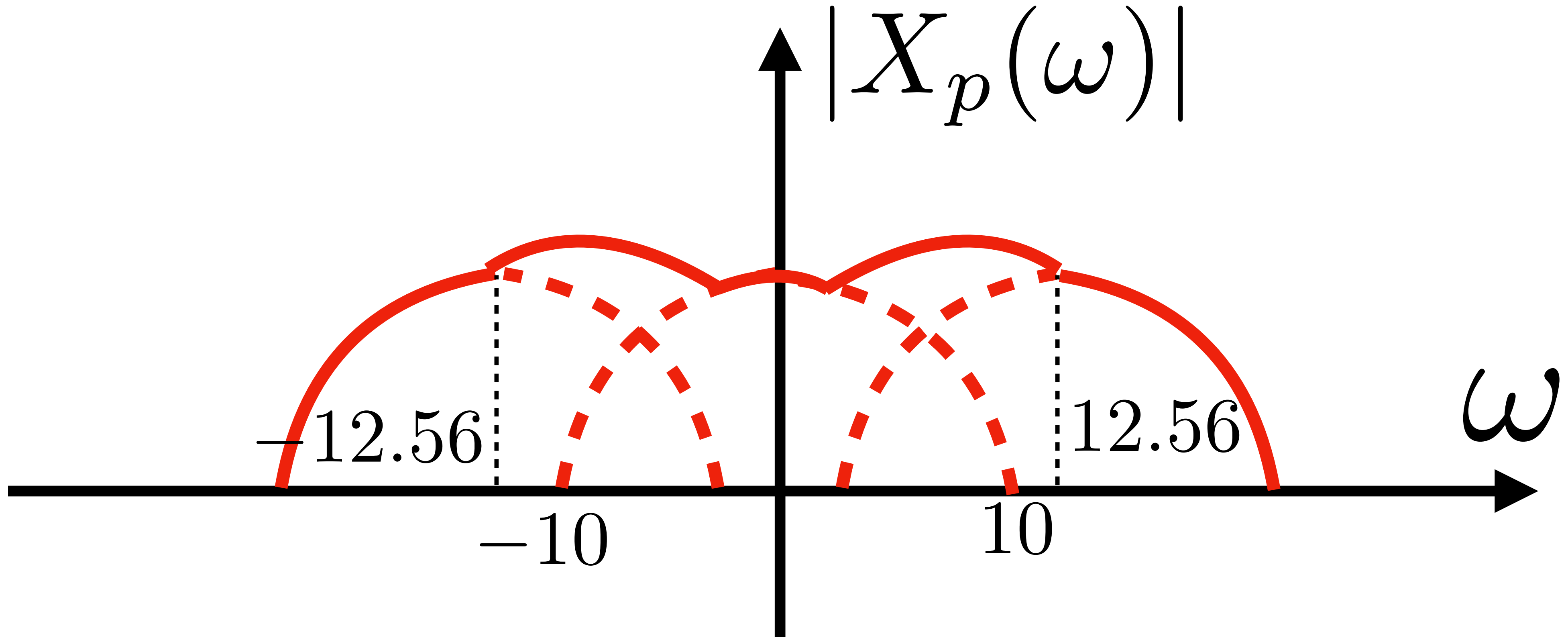
$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

with T=0.5 sec.

Example 5

(in the exam, we have to explain every step very well… with comments and explanations…)

It appears “replicas” each $\omega_s = \dfrac{2\pi}{T} = 12.56$ rad/sec :



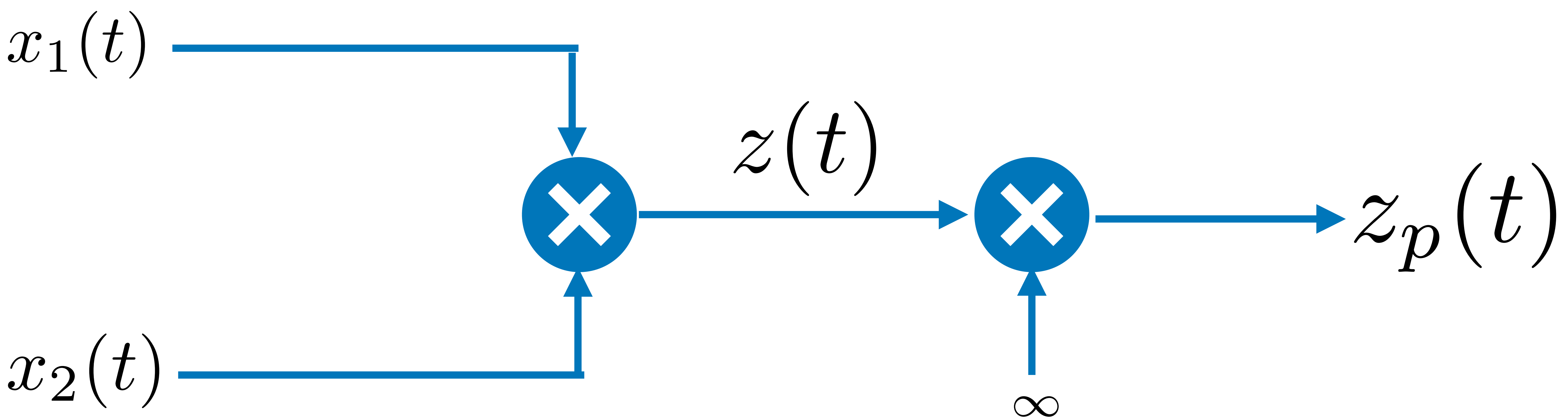
Note that these are just the first replicas (there are an infinite number of replicas)

there is “Aliasing” in this case !!! we lose information….

we cannot recover the original signal x(t)…

Example 6

Let consider the following system below:



$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

both input signals are band-limited, i.e.,

$$X_1(\omega) \geq 0 \qquad |\omega| \geq \omega_1$$

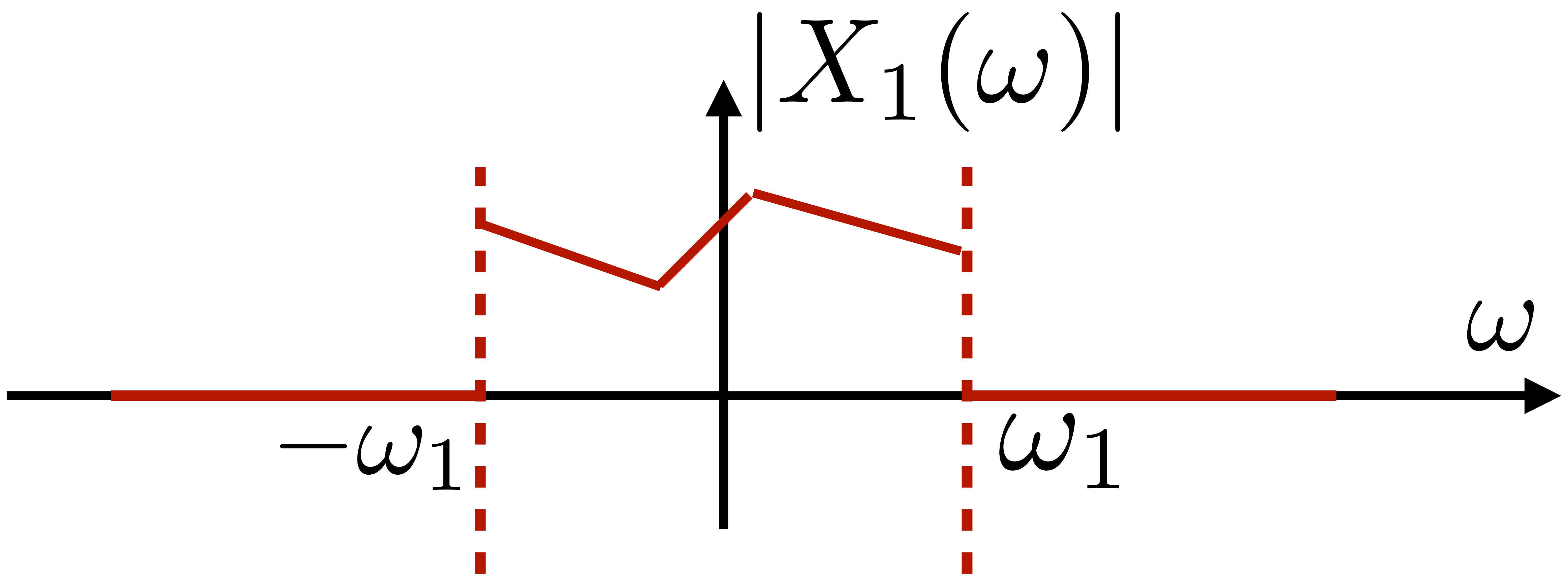
$$X_2(\omega) \geq 0 \qquad |\omega| \geq \omega_2$$

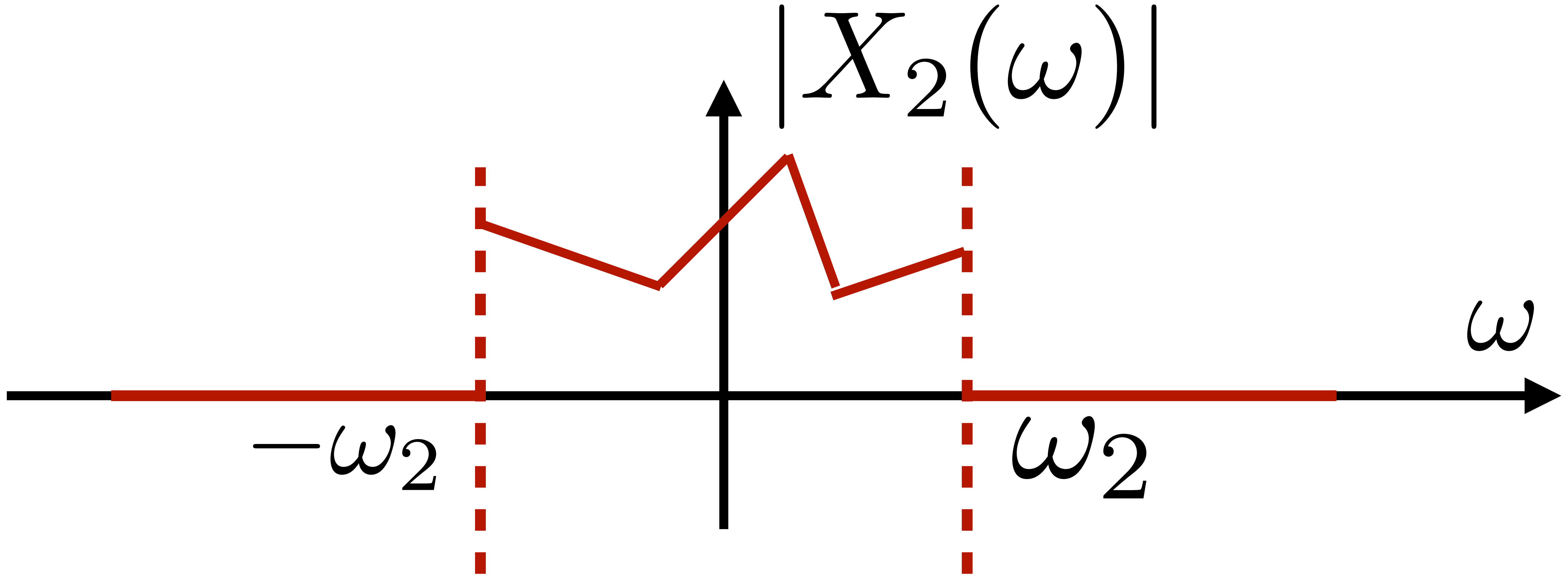
Find the maximum T such that z(t) can be recovers from z_p(t) with an ideal band-pass filter.

Example 6

$$z(t) = x_1(t)x_2(t)$$

$$Z(\omega) = X_1(\omega) * X_2(\omega)$$





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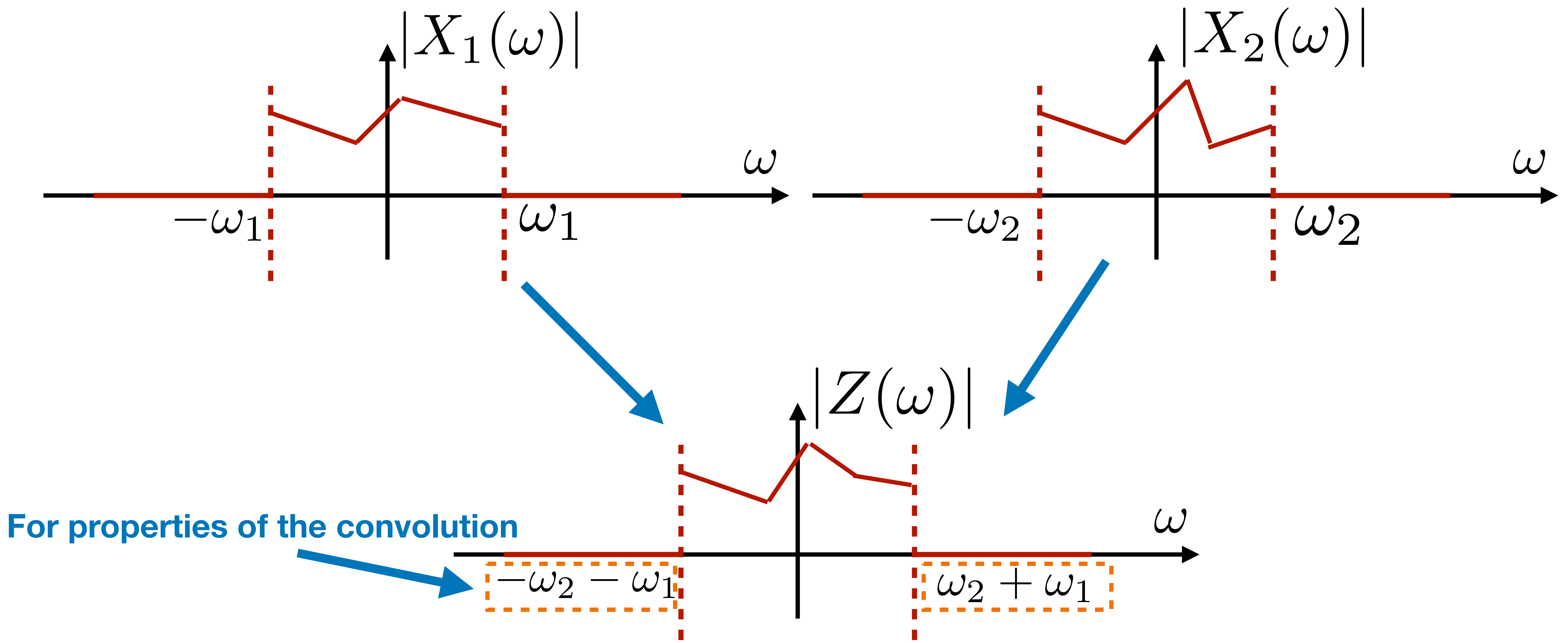
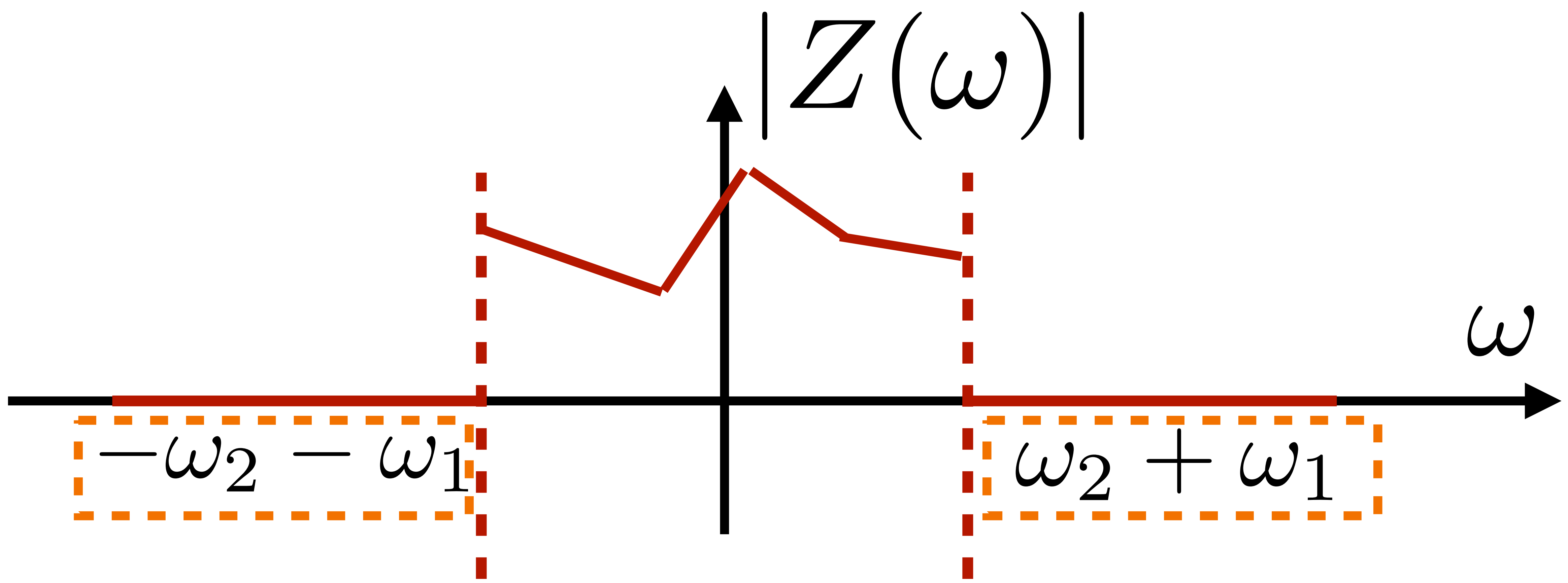


Example 6



Applying Nyquist:

$$W = \omega_2 + \omega_1$$

$$\omega_s \geq 2(\omega_2 + \omega_1)$$

$$T \leq \frac{\pi}{\omega_2 + \omega_1} \longrightarrow T_{\mathtt{max}} = \frac{\pi}{\omega_2 + \omega_1}$$

This is the solution.

Example 7

Let consider the following system below:

$$z(t) = x_1(t) * x_2(t) \longrightarrow \otimes \longrightarrow z_p(t)$$

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

both input signals are band-limited, i.e.,

$$X_1(\omega) \geq 0 \qquad |\omega| \geq \omega_1$$
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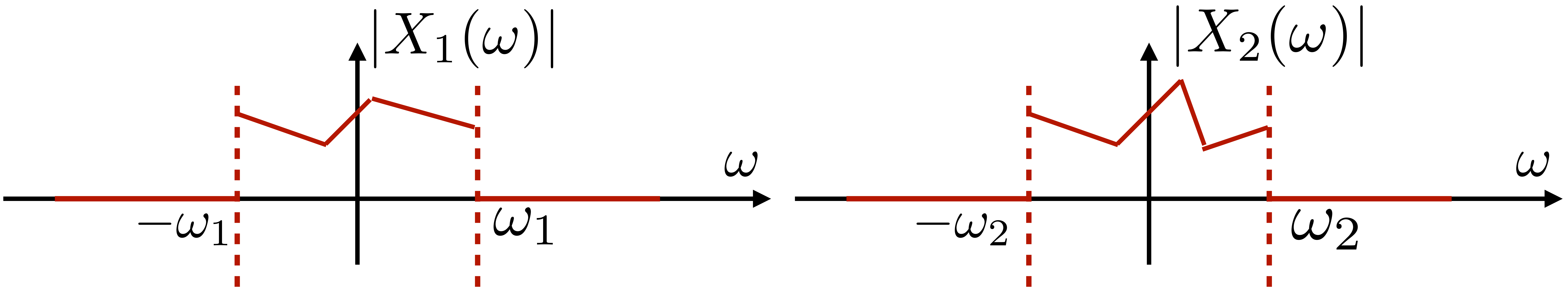

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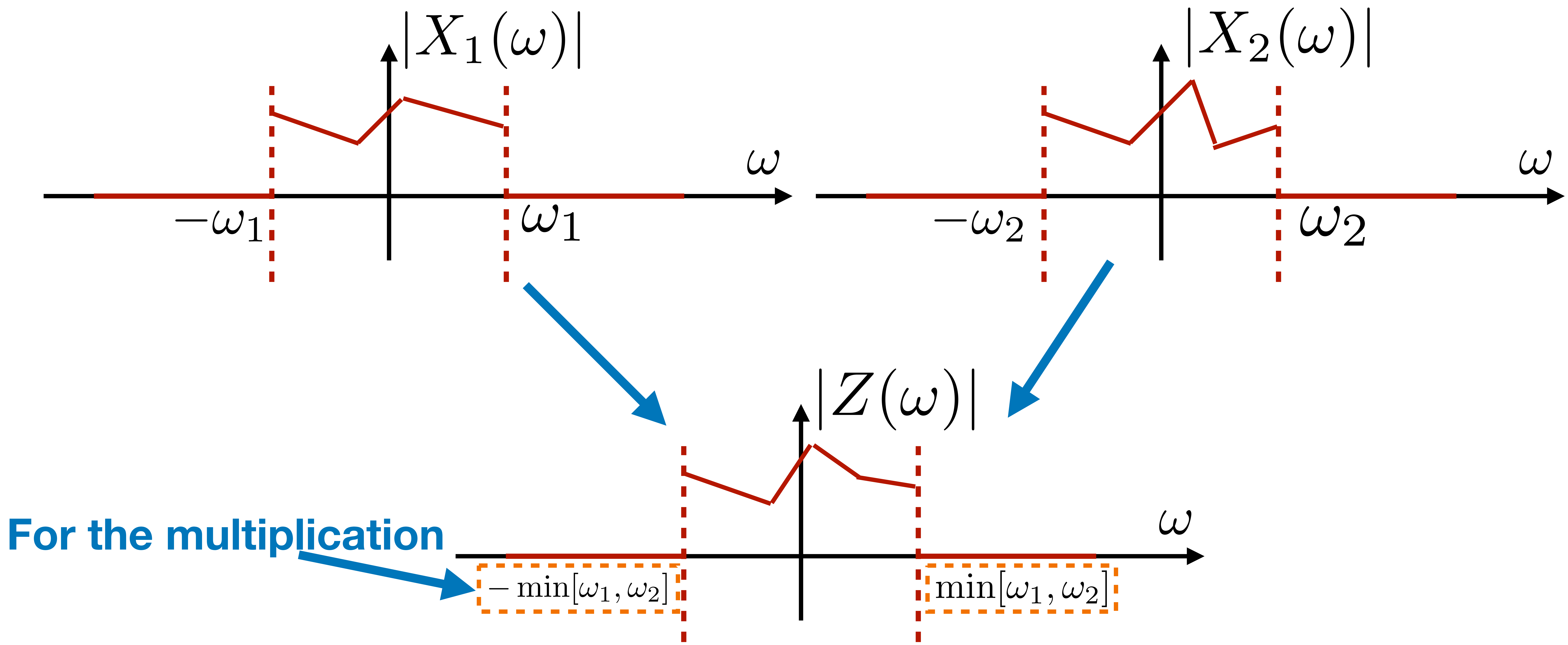
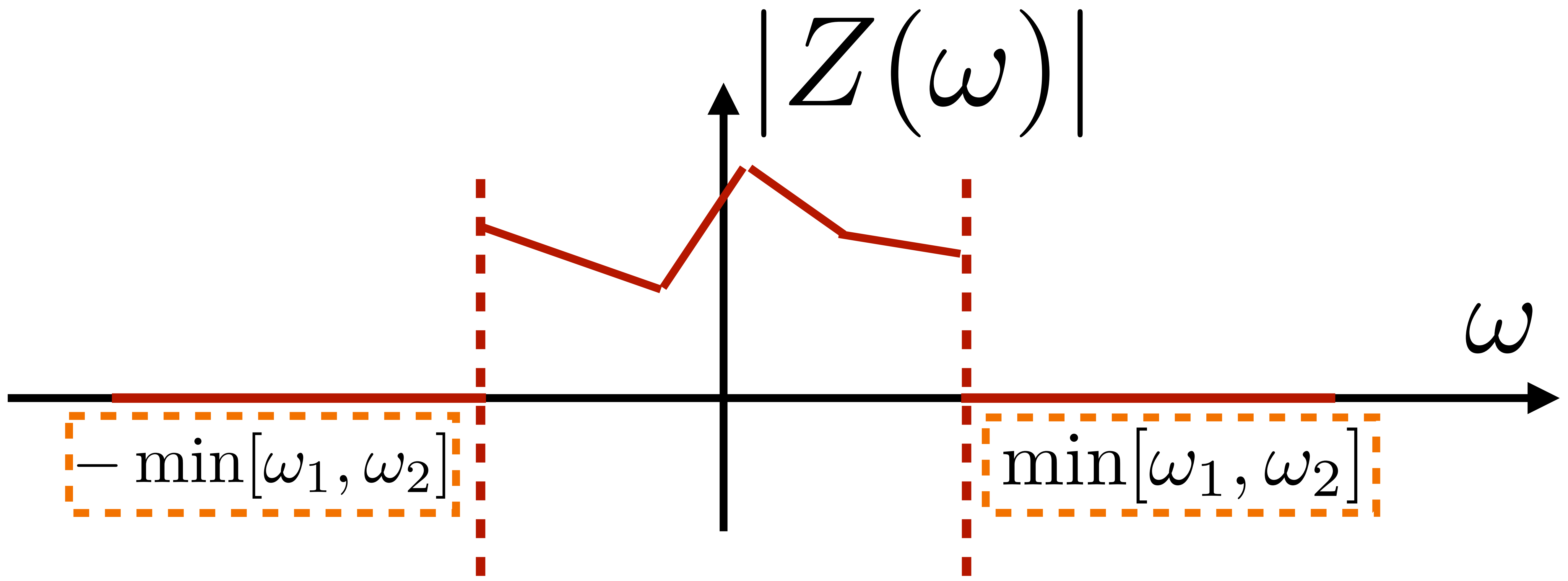


Example 7



Applying Nyquist:

$$W = \min[\omega_1, \omega_2]$$

$$\omega_s \geq 2\min[\omega_1, \omega_2]$$

$$T \leq \frac{\pi}{\min[\omega_1, \omega_2]} \longrightarrow T_{\texttt{max}} = \frac{\pi}{\min[\omega_1, \omega_2]}$$

This is the solution.

Example 8

A signal x(t) has been *perfectly* recovered from a sequence x[n], using a *sampling rate* of $\omega_s = 10000\pi$

Find or obtain some inequality regarding the W such that

$$X(\omega) \geq 0 \quad |\omega| \leq W$$

$$W = ?$$

Example 8

Applying Nyquist:

$$\omega_s \geq 2W$$

$$W \leq \frac{\omega_s}{2}$$

$$W \leq 5000\pi$$

This is the solution.

Example 9

Write the Nyquest's inequalities for the following signal:

$$x(t) = 1 + \cos(2000\pi t) + \sin(4000\pi t)$$

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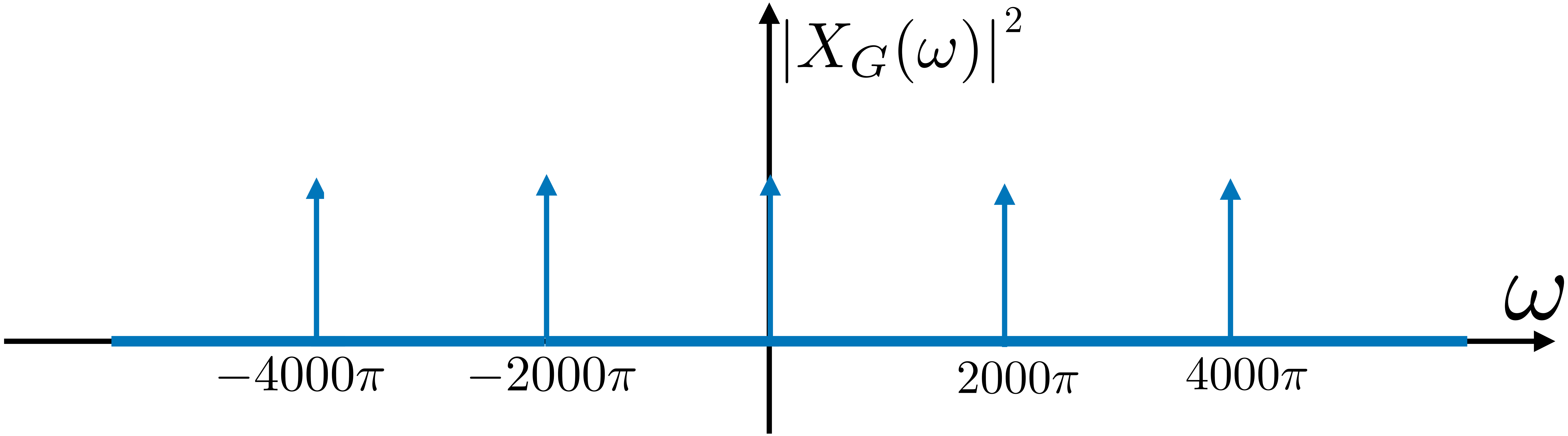
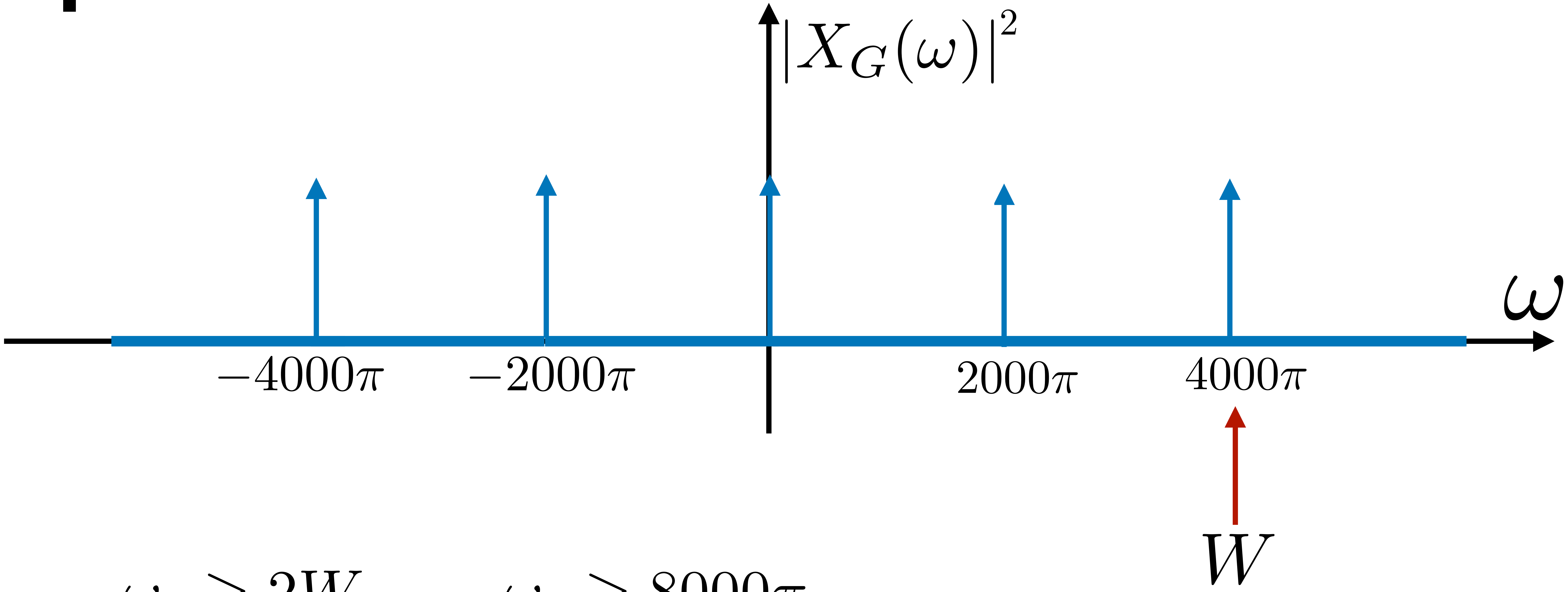
$$x(t) = 1 + \cos(2000\pi t) + \sin(4000\pi t)$$

Generalized Fourier Transform:

$$X_G(\omega) = 2\pi \left(\delta(\omega) + \frac{1}{2}\left(\delta(\omega - 2000\pi) + \delta(\omega + 2000\pi)\right) + \frac{1}{2j}\left(\delta(\omega - 4000\pi) - \delta(\omega + 4000\pi)\right)\right)$$



Example 9



$$\omega_s \geq 2W, \qquad \omega_s \geq 8000\pi,$$

$$T \leq \frac{\pi}{W}, \quad T \leq \frac{1}{4000} \quad \text{sec}$$

This is the solution.

Example 10

Write the Nyquest's inequalities for the following signal:

$$x(t) = 1 + \cos(2000\pi t) + \sin(2000\pi t)$$

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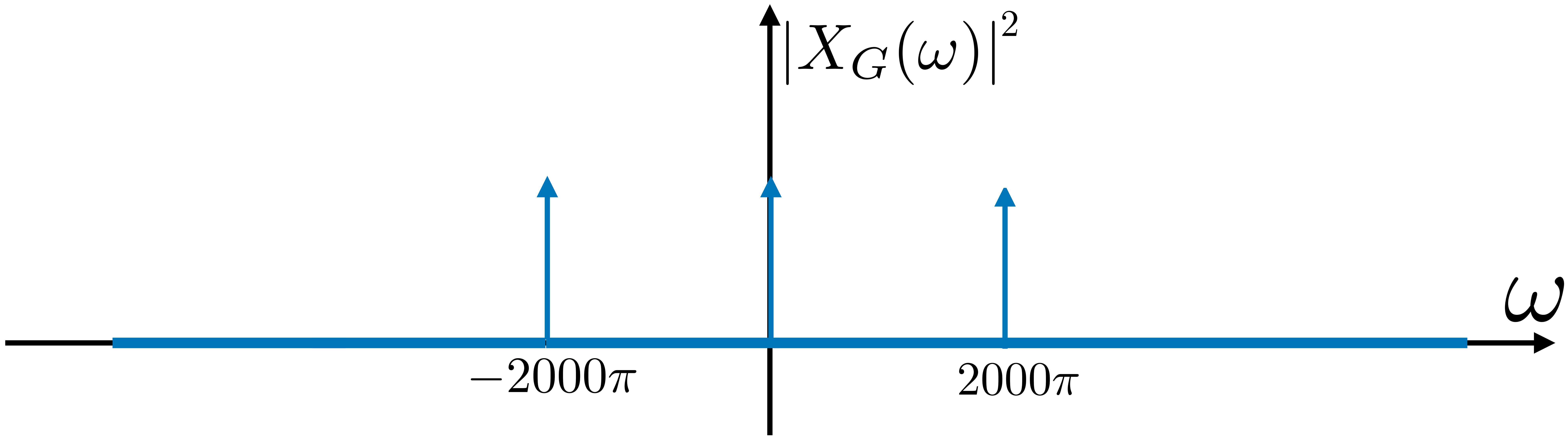
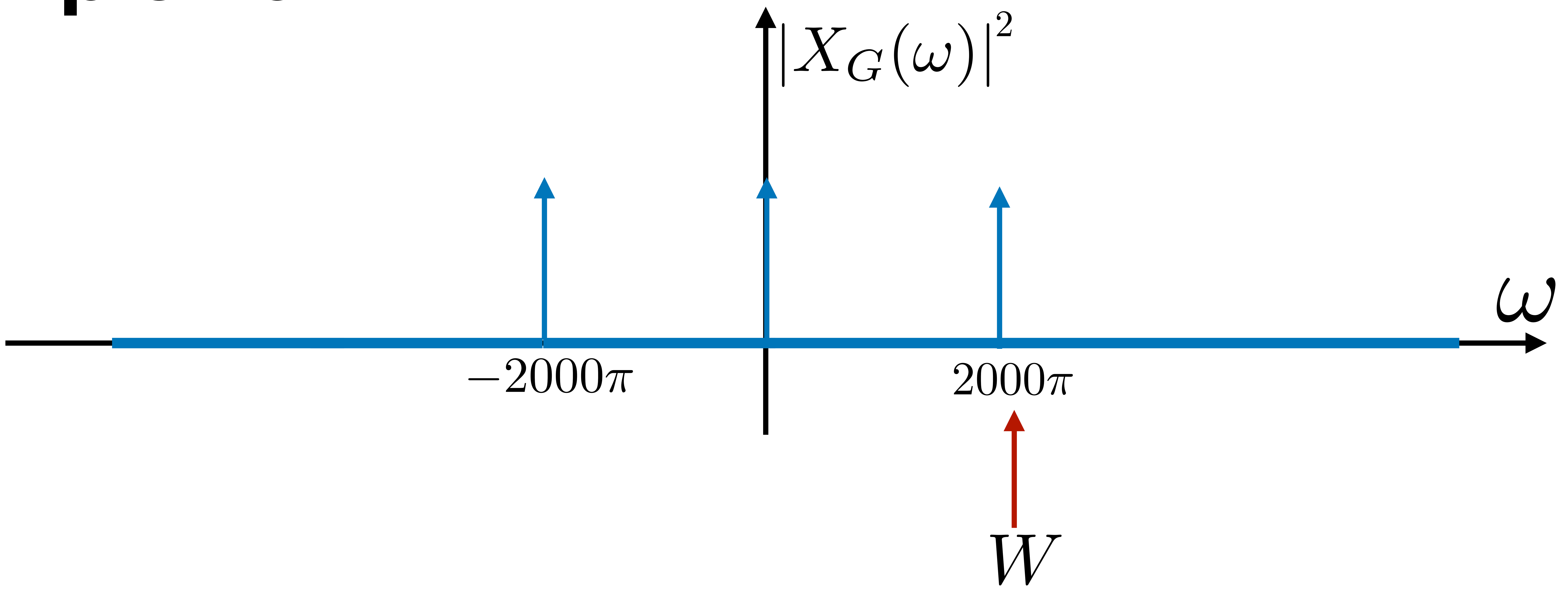
Generalized Fourier Transform:

$$X_G(\omega) = 2\pi \left(\delta(\omega) + \frac{1}{2} \left(\delta(\omega - 2000\pi) + \delta(\omega + 2000\pi)\right) + \frac{1}{2j} \left(\delta(\omega - 2000\pi) - \delta(\omega + 2000\pi)\right)\right)$$



Example 10



$$\omega_s \geq 2W, \qquad \omega_s \geq 4000\pi,$$

$$T \leq \frac{\pi}{W}, \quad T \leq \frac{1}{2000} \quad \text{sec}$$

This is the solution.

Example 11

Write the Nyquest's inequalities for the following signal:

$$x(t) = 1 + \cos(4000\pi t) + \sin(2000\pi t)$$

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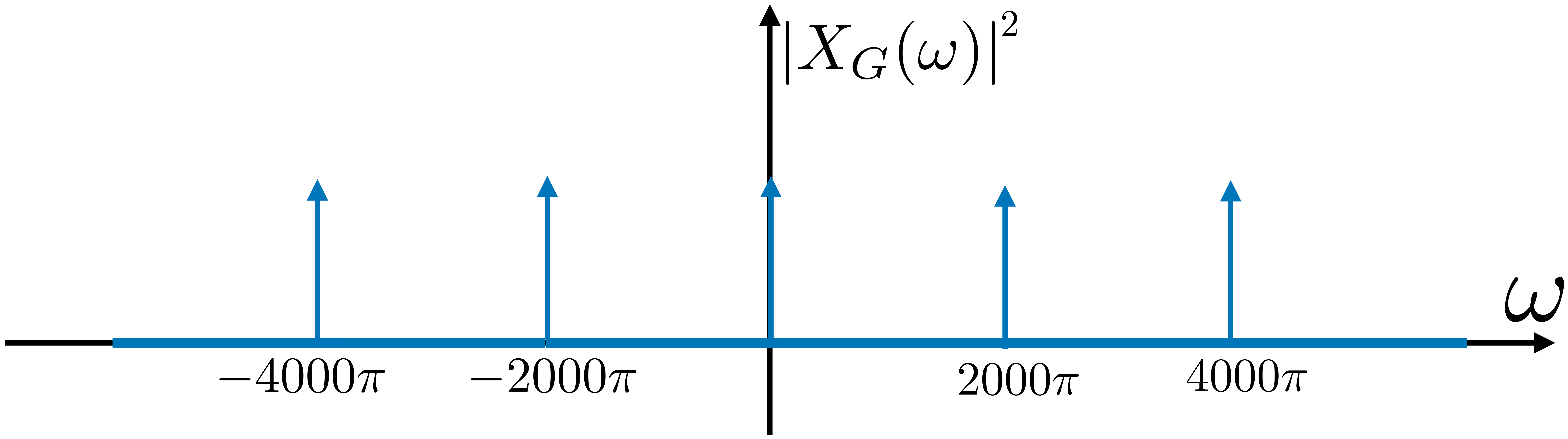
$$x(t) = 1 + \cos(4000\pi t) + \sin(2000\pi t)$$

Generalized Fourier Transform:

$$X_G(\omega) = 2\pi\left(\delta(\omega) + \frac{1}{2}\left(\delta(\omega - 4000\pi) + \delta(\omega + 4000\pi)\right) + \frac{1}{2j}\left(\delta(\omega - 2000\pi) - \delta(\omega + 2000\pi)\right)\right)$$



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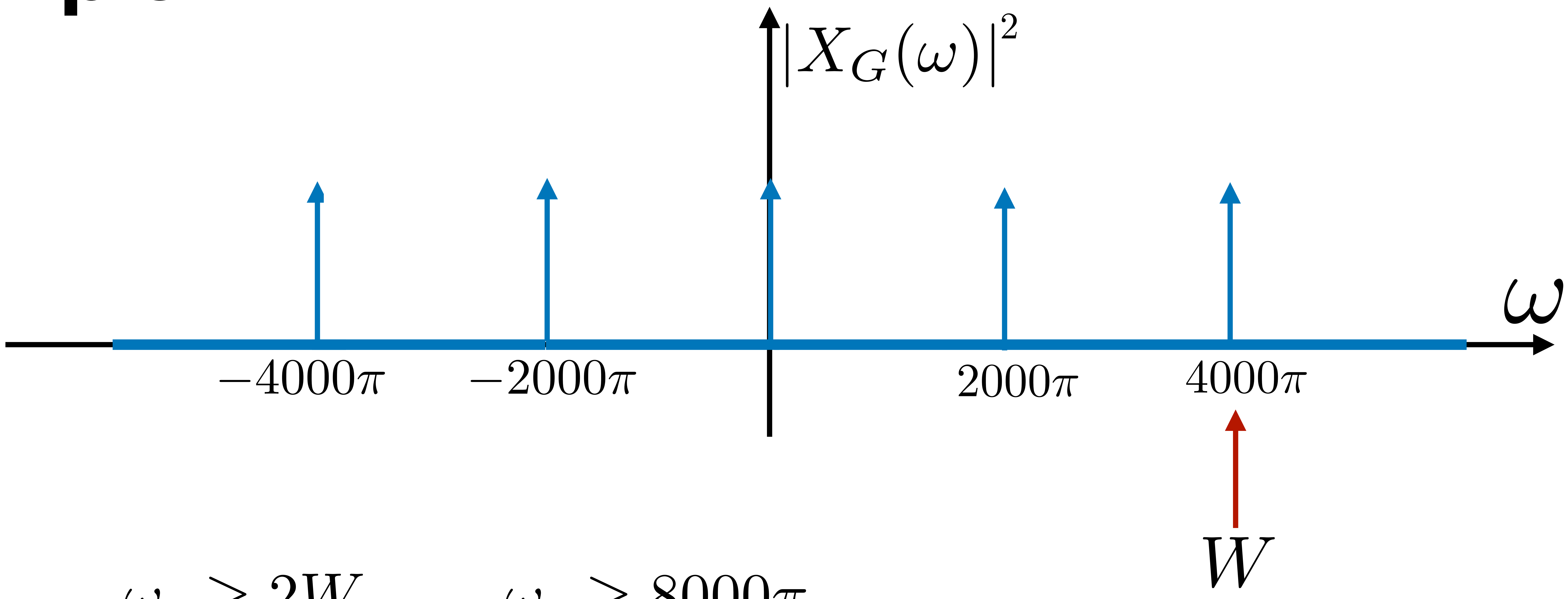

$$\omega_s \geq 2W, \qquad \omega_s \geq 8000\pi,$$

$$T \leq \frac{\pi}{W}, \quad T \leq \frac{1}{4000} \quad \text{sec}$$

This is the solution.

Questions?