Solved Problems - DFT

Linear systems and circuit applications

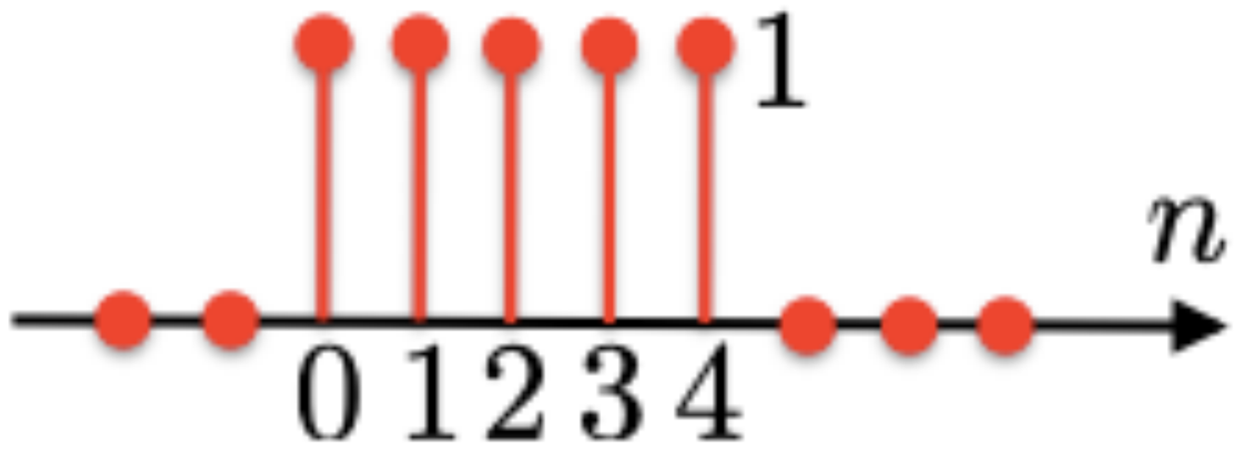
Discrete Time Systems

Luca Martino — luca.martino@urjc.es — http://www.lucamartino.altervista.org
Laura Cornejo — laura.cornejo@urjc.es

Example 1

Let consider the following signal:

$$x[n] = \begin{cases} 1 & n = 0, \ldots, 4 \\ 0 & n \leq -1 \text{ o } n \geq 5 \end{cases}$$



(a) Compute the FT of x[n]

(b) Compute the coefficients a_k of the Fourier Series of the “periodic brother” signal $\widetilde{x}[n]$ obtained repeating the x[0]=1, x[1]=1, x[2]=1, x[3]=1, and x[4]=1, each N=5 and N=6 times steps.

(c) Considering the data x[0]=1, x[1]=1, x[2]=1, x[3]=1, and x[4]=1, say what is the value of L, and considering N=6 compute the DFT.

(d) Compare the a_k with the DFT.

Example 1

(a) We can write:

$$x[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4]$$

We can compute the FT in two (equivalent) ways: first of all,

$$\delta[n-n_0] \longleftrightarrow e^{-j\Omega n_0}$$

$$X(\Omega) = 1 + e^{-j\Omega} + e^{-j2\Omega} + e^{-j3\Omega} + e^{-j4\Omega}$$

Example 1

$$X(\Omega) = 1 + e^{-j\Omega} + e^{-j2\Omega} + e^{-j3\Omega} + e^{-j4\Omega}$$

or in this way:

$$X(\Omega) = \sum_{n=-\infty}^{+\infty} x[n] \cdot e^{-j\Omega n} = \sum_{\boxed{n=0}}^{\boxed{4}} e^{-j\Omega n} = \frac{1 - e^{-j5\Omega}}{1 - e^{-j\Omega}} = \frac{\sin(5\Omega/2)}{\sin(\Omega/2)} e^{-j2\Omega}$$

$$X(0) = 5$$

using this sum formula (with N2=4 and N1=0):

$$\sum_{n=N_1}^{N_2} r^n = r^{N_1} \frac{1 - r^{N_2 - N_1 + 1}}{1 - r}$$

Example 1

These two formulas are equivalent:

$$X(\Omega) = 1 + e^{-j\Omega} + e^{-j2\Omega} + e^{-j3\Omega} + e^{-j4\Omega}$$

$$X(\Omega) = \frac{1 - e^{-j5\Omega}}{1 - e^{-j\Omega}} \quad \text{and} \quad X(0) = 5$$

Indeed, in both cases, as examples we have: (this is not required in the problem)

$$X(0) = 1 \qquad X(0.2) = 4.4232 - 1.8701j \qquad X(0.5) = 2.0725 - 3.2277j$$

$$X(1) = -0.5195 - 1.1351j \qquad X\left(\frac{\pi}{2}\right) = X(\pi) = 1$$

Example 1

(b) We will use the following formulas:

$$X(\Omega) = \frac{1 - e^{-j5\Omega}}{1 - e^{-j\Omega}}$$

the coefficients can be easily computed as:

$$\boxed{X_k = a_k = \frac{1}{N} X(k\Omega_0) = \frac{1}{N} X\left(k\frac{2\pi}{N}\right)}$$

$$\boxed{\Omega_0 = \frac{2\pi}{N}} \qquad a_k = a_{k+N}$$

for N=5:

$$a_k = \frac{1}{5}\frac{1 - e^{-j5k\frac{2\pi}{5}}}{1 - e^{-jk\frac{2\pi}{5}}} = \frac{1}{5}\frac{1 - e^{-jk2\pi}}{1 - e^{-jk\frac{2\pi}{5}}} = 0 \qquad k \neq 0$$

$$k = 1, 2, 3, 4$$

$$a_k = a_{k+5}$$

Example 1

for N=5:

$$a_k = \frac{1}{5}\frac{1 - e^{-jk2\pi}}{1 - e^{-jk\frac{2\pi}{5}}} = 0 \quad k \neq 0$$

for k=0, we can have an indeterminate form, which can be easily solved in this way (from the definition of a_k with k=0) :

$$a_0 = \frac{1}{N}\sum_{n=0}^{N-1} x[n] = \frac{1}{5}5 = 1$$

then the coefficients of the FS for N=5 are ("summary formula"):

$$a_k = \delta[k] \quad \text{and} \quad a_k = a_{k+5}$$

Example 1

for N=6:

$$X(\Omega) = \frac{1 - e^{-j5\Omega}}{1 - e^{-j\Omega}}$$

the coefficients can be easily computed as:

$$X_k = a_k = \frac{1}{N} X(k\Omega_0) = \frac{1}{N} X\left(k \frac{2\pi}{N}\right)$$

$$\Omega_0 = \frac{2\pi}{N} \qquad a_k = a_{k+N}$$

$$a_k = \frac{1}{6} \frac{1 - e^{-jk\frac{5\pi}{3}}}{1 - e^{-jk\frac{\pi}{3}}} \quad k \neq 0 \qquad a_0 = \frac{1}{N} \sum_{n=0}^{N-1} x[n] = \frac{1}{6} 5 = \frac{5}{6}$$

$$\text{and} \quad a_k = a_{k+6}$$

Example 1

c) First of all, L=5.

There are several ways to compute the DFT. In this problem, the easiest and fastest way is using the following formulas:

$$X(\Omega) = \frac{1 - e^{-j5\Omega}}{1 - e^{-j\Omega}}$$

$$\boxed{X_N[k] = X(\Omega)\Big|_{k\frac{2\pi}{N}} = X\left(k\frac{2\pi}{N}\right)}$$

$$\boxed{X_N[k] = X(\Omega)\Big|_{k\Omega_0} = X\left(k\Omega_0\right)}$$

$$\Omega_0 = \frac{2\pi}{N}$$

for N=6:

$$X_6[k] = \frac{1 - e^{-jk\frac{5\pi}{3}}}{1 - e^{-jk\frac{\pi}{3}}} \quad k \neq 0$$

$$X_6[0] = \sum_{n=0}^{N-1} x[n] = 5$$

Example 1

d) we find that:

$$a_k = \frac{1}{6} X_6[k]$$

Example 2

Let consider the following signal:

$$x[0] = 3, x[1] = -5, x[2] = 1.5, \quad x[n] = 0 \quad \text{for the rest of n}$$

(a) Say what is L.

(b) Compute the FT of x[n].

(c) considering N= 6, 7, 8 compute the DFT X_N[k].

(d) Say what is the output of the Matlab function fft(x,N) with N=6, and what are the frequency associated to each elements of the output.

Example 2

(a) L=3

(b) The FT ca be easily obtained in two equivalent ways:

$$x[n] = 3 - 5\delta[n-1] + 1.5\delta[n-2]$$

$$X(\Omega) = 3 - 5e^{-j\Omega} + 1.5e^{-j2\Omega}$$

Example 2

(c) There are several ways to compute the DFT. In this problem, the easiest and fastest way is using the following formulas:

$$\boxed{X_N[k] = X(\Omega)\Big|_{k\frac{2\pi}{N}} = X\left(k\frac{2\pi}{N}\right)}$$

$$\boxed{X_N[k] = X(\Omega)\Big|_{k\Omega_0} = X\left(k\Omega_0\right)}$$

$$\Omega_0 = \frac{2\pi}{N}$$

$$X(\Omega) = 3 - 5e^{-j\Omega} + 1.5e^{-j2\Omega}$$

$$X_N[k] = 3 - 5e^{-jk\frac{2\pi}{N}} + 1.5e^{-j2k\frac{2\pi}{N}}$$

Example 2

$$X_N[k] = 3 - 5e^{-jk\frac{2\pi}{N}} + 1.5e^{-j2k\frac{2\pi}{N}}$$

$$X_N[k] = 3 - 5e^{-jk\frac{2\pi}{N}} + 1.5e^{-jk\frac{4\pi}{N}}$$

N=6:

$$X_6[k] = 3 - 5e^{-jk\frac{\pi}{3}} + 1.5e^{-jk\frac{2\pi}{3}}$$

N=7:

$$X_7[k] = 3 - 5e^{-jk\frac{2\pi}{7}} + 1.5e^{-jk\frac{4\pi}{7}}$$

N=8:

$$X_8[k] = 3 - 5e^{-jk\frac{\pi}{4}} + 1.5e^{-jk\frac{\pi}{2}}$$

Example 2

$$X_6[k] = 3 - 5e^{-jk\frac{\pi}{3}} + 1.5e^{-jk\frac{2\pi}{3}}$$

(d) The Matlab function fft(x,N) returns N values.

Matlab returns N complex numbers:

$$X_N[0], X_N[1], X_N[2] \ldots, X_N[N-1]$$

frequencies $\longrightarrow$ $0 \quad \Omega_0 \quad 2\Omega_0 \quad (N-1)\Omega_0$

$$\boxed{\Omega_0 = \frac{2\pi}{N}}$$

Example 2

Then in this case:

$$X_6[k] = 3 - 5e^{-jk\frac{\pi}{3}} + 1.5e^{-jk\frac{2\pi}{3}}$$

$$X_6[0] = -0.5$$

$$X_6[1] = 3 - 5e^{-j\frac{\pi}{3}} + 1.5e^{-j\frac{2\pi}{3}} = -0.2500 + 3.0311j$$

$$X_6[2] = 3 - 5e^{-j\frac{2\pi}{3}} + 1.5e^{-j\frac{4\pi}{3}} = 4.7500 + 5.6292j$$

$$X_6[3] = 3 - 5e^{-j\pi} + 1.5e^{-j2\pi} = 9.5$$

$$X_6[4] = 3 - 5e^{-jk\frac{4\pi}{3}} + 1.5e^{-jk\frac{8\pi}{3}} = 4.7500 - 5.6292j$$

$$X_6[5] = 3 - 5e^{-jk\frac{5\pi}{3}} + 1.5e^{-jk\frac{10\pi}{3}} = -0.2500 - 3.0311j$$

Example 2

$$\Omega_0 = \frac{2\pi}{6} = \frac{\pi}{3}$$

$$X_6[0] = -0.5 \longrightarrow \Omega = 0$$

$$X_6[1] = 3 - 5e^{-j\frac{\pi}{3}} + 1.5e^{-j\frac{2\pi}{3}} = -0.2500 + 3.0311j \longrightarrow \Omega = 1\Omega_0 = \frac{\pi}{3}$$

$$X_6[2] = 3 - 5e^{-j\frac{2\pi}{3}} + 1.5e^{-j\frac{4\pi}{3}} = 4.7500 + 5.6292j \longrightarrow \Omega = 2\Omega_0 = \frac{2\pi}{3}$$

$$X_6[3] = 3 - 5e^{-j\pi} + 1.5e^{-j2\pi} = 9.5 \longrightarrow \Omega = 3\Omega_0 = \frac{3\pi}{3} = \pi$$

$$X_6[4] = 3 - 5e^{-jk\frac{4\pi}{3}} + 1.5e^{-jk\frac{8\pi}{3}} = 4.7500 - 5.6292j \longrightarrow \Omega = 4\Omega_0 = \frac{4\pi}{3}$$

$$X_6[5] = 3 - 5e^{-jk\frac{5\pi}{3}} + 1.5e^{-jk\frac{10\pi}{3}} = -0.2500 - 3.0311j$$

$$\searrow \Omega = 5\Omega_0 = \frac{5\pi}{3}$$

Example 2

Just to see the matlab view and check that what we wrote is right:

```
>> x=[3 -5 1.5];
>> output=fft(x,6)

output =

  Columns 1 through 5

  -0.5000 + 0.0000i  -0.2500 + 3.0311i   4.7500 + 5.6292i   9.5000 + 0.0000i   4.7500 - 5.6292i

  Column 6

  -0.2500 - 3.0311i
```

Example 2

Just to see the matlab view and check that what we wrote is right:

```
>> 3-5*exp(-j*pi/3)+1.5*exp(-j*2*pi/3)

ans =

  -0.2500 + 3.0311i

>> 3-5*exp(-j*2*pi/3)+1.5*exp(-j*4*pi/3)

ans =

   4.7500 + 5.6292i
```

```
>> 3-5*exp(-j*pi)+1.5*exp(-j*2*pi)

ans =

   9.5000 + 0.0000i

>> 3-5*exp(-j*4*pi/3)+1.5*exp(-j*8*pi/3)

ans =

   4.7500 - 5.6292i

>> 3-5*exp(-j*5*pi/3)+1.5*exp(-j*10*pi/3)

ans =

  -0.2500 - 3.0311i
```

Example 3

Let consider the following signal:

$$x[0] = 10, x[1] = -5, \quad x[n] = 0 \quad \text{for the rest of n}$$

(a) Say what is L.

(b) Compute the FT of x[n].

(c) considering a generic N compute the DFT X_N[k].

(d) Say what is the output of the Matlab function fft(x,N) with N=2,3,4,5,6,7,8, and what are the frequency associated to each elements of the output.

In an exam, you have to write all the "complete" formulas with exponentials etc. here I will give you the numerical results in order to check your calculus…

Example 3

$$x[0] = 10, x[1] = -5, \quad x[n] = 0 \quad \text{for the rest of n}$$

(a) L=2

(b) The FT is

$$x[n] = 10 - 5\delta[n-1]$$

$$X(\Omega) = 10 - 5e^{-j\Omega}$$

Example 3

$$X(\Omega) = 10 - 5e^{-j\Omega}$$

(c) The DFT is

$$X_N[k] = 10 - 5e^{-jk\frac{2\pi}{N}}$$

Example 3

$$X_N[k] = 10 - 5e^{-jk\frac{2\pi}{N}}$$

(d) N=2

$$\Omega_0 = \frac{2\pi}{N} = \frac{2\pi}{2} = \pi$$

$$X_2[0] = 10 - 5 = 5 \longrightarrow \Omega = 0\Omega_0 = 0$$

$$X_2[1] = 10 - 5e^{-j\pi} = 10 + 5 = 15 \longrightarrow \Omega = 1\Omega_0 = \pi$$

```
>> x=[10 -5];
>> fft(x,2)

ans =

     5    15
```

```
>> Omega_0=pi

Omega_0 =

    3.1416

>> Omega=[0:1]*Omega_0

Omega =

         0    3.1416
```

Example 3

$$X_N[k] = 10 - 5e^{-jk\frac{2\pi}{N}}$$

(d) N=3

In an exam, you have to write all the "complete" formulas with exponentials etc. here I will give you the numerical results in order to check your calculus... as in the previous slide

$$\Omega_0 = \frac{2\pi}{N} = \frac{2\pi}{3}$$

```
>> x=[10 -5];
>> fft(x,3)

ans =

   5.0000 + 0.0000i  12.5000 + 4.3301i  12.5000 - 4.3301i
```

```
>> Omega_0=2*pi/3

Omega_0 =

    2.0944

>> Omega=[0:2]*Omega_0

Omega =

         0    2.0944    4.1888
```

Example 3

(d) N=4

In an exam, you have to write all the “complete” formulas with exponentials etc. here I will give you the numerical results in order to check your calculus…

```
here the FFT:

ans =

   5.0000 + 0.0000i  10.0000 + 5.0000i  15.0000 + 0.0000i  10.0000 - 5.0000i

Omega_0:

Omega_0 =

    1.5708

Frequencies:

Omega =

         0    1.5708    3.1416    4.7124
```

Example 3

(d) N=5

In an exam, you have to write all the “complete” formulas with exponentials etc. here I will give you the numerical results in order to check your calculus…

```
here the FFT:

ans =

   5.0000 + 0.0000i   8.4549 + 4.7553i  14.0451 + 2.9389i  14.0451 - 2.9389i   8.4549 - 4.7553i

Omega_0:

Omega_0 =

    1.2566

Frequencies:

Omega =

         0    1.2566    2.5133    3.7699    5.0265
```

Example 3

(d) N=6

In an exam, you have to write all the "complete" formulas with exponentials etc. here I will give you the numerical results in order to check your calculus...

```
here the FFT:

ans =

  Columns 1 through 5

   5.0000 + 0.0000i   7.5000 + 4.3301i  12.5000 + 4.3301i  15.0000 + 0.0000i  12.5000 - 4.3301i

  Column 6

   7.5000 - 4.3301i

Omega_0:

Omega_0 =

    1.0472

Frequencies:

Omega =

         0    1.0472    2.0944    3.1416    4.1888    5.2360
```

Example 3

(d) N=7

In an exam, you have to write all the "complete" formulas with exponentials etc. here I will give you the numerical results in order to check your calculus…

```
here the FFT:

ans =

  Columns 1 through 5

   5.0000 + 0.0000i   6.8826 + 3.9092i  11.1126 + 4.8746i  14.5048 + 2.1694i  14.5048 - 2.1694i

  Columns 6 through 7

  11.1126 - 4.8746i   6.8826 - 3.9092i

Omega_0:

Omega_0 =

    0.8976

Frequencies:

Omega =

         0    0.8976    1.7952    2.6928    3.5904    4.4880    5.3856
```

Example 3

(d) N=8

In an exam, you have to write all the "complete" formulas with exponentials etc. here I will give you the numerical results in order to check your calculus...

```
here the FFT:

ans =

  Columns 1 through 4

   5.0000 + 0.0000i   6.4645 + 3.5355i  10.0000 + 5.0000i  13.5355 + 3.5355i

  Columns 5 through 8

  15.0000 + 0.0000i  13.5355 - 3.5355i  10.0000 - 5.0000i   6.4645 - 3.5355i

Omega_0:

Omega_0 =

    0.7854

Frequencies:

Omega =

         0    0.7854    1.5708    2.3562    3.1416    3.9270    4.7124    5.4978
```

Example 4

Let consider the following signal:

$$x[0] = 10, x[1] = -5, \quad x[n] = 0 \quad \text{for the rest of n}$$

say what is L and if it is possible to compute the DFT for N=1.

Example 4

$L=2$ and it is not possible to compute DFT for $N<L$ ($N=1$ for instance) since the result has no meaning (el resultado no tiene ningún sentido)

Example 5

Let consider the following signal:

$$x[0] = 10, x[1] = -5, \quad x[n] = 0 \quad \text{for the rest of n}$$

Choose a proper N such that the DFT X_N[k] returns you, for some k, the value of the FT X(Omega) at Omega=pi/2.

Example 5

We know that: $$\Omega_0 = \frac{2\pi}{N}$$

and the frequencies are:

$$\Omega^{(k)} = k\Omega_0 = k\frac{2\pi}{N}$$

Example 5

then one possibility is to consider N=4 and k=1: (we can have several possibilities)

$$\Omega^{(k)} = k\frac{2\pi}{4} = k\frac{\pi}{2}$$

$$\Downarrow$$

$$k = 1 \implies \frac{\pi}{2}$$

Example 5

(d) N=4

In an exam, you have to write all the "complete" formulas with exponentials etc. here I will give you the numerical results in order to check your calculus...

```
here the FFT:

ans =

   5.0000 + 0.0000i  10.0000 + 5.0000i  15.0000 + 0.0000i  10.0000 - 5.0000i

Omega_0:

Omega_0 =

    1.5708

Frequencies:

Omega =

         0    1.5708    3.1416    4.7124
```

THIS IS JUST TO CHECK...

$$X(\Omega = \pi/2) = X(\pi/2)$$

$$k = 1 \Longrightarrow \frac{\pi}{2}$$

Example 5

JUST TO CHECK: $$X(\Omega) = 10 - 5e^{-j\Omega}$$

$$X(\Omega = \pi/2) = X(\pi/2) = 10 - 5e^{-j\frac{\pi}{2}}$$

$$X\left(\frac{\pi}{2}\right) = 10 - 5\left(\cos\left(\frac{\pi}{2}\right) - j\sin\left(\frac{\pi}{2}\right)\right)$$

$$X\left(\frac{\pi}{2}\right) = 10 - 5\left(0 - j1\right) = 10 + 5j$$ **correct !!!**

Example 6

Let consider the following signal:

$$x[0] = 10, x[1] = -5, \quad x[n] = 0 \quad \text{for the rest of n}$$

Choose a proper N such that the DFT X_N[k] returns you, for some k, the value of the FT X(Omega) at Omega=2pi/3.

Example 6

We know that:

$$\Omega_0 = \frac{2\pi}{N}$$

and the frequencies are:

$$\Omega^{(k)} = k\Omega_0 = k\frac{2\pi}{N}$$

Example 6

then one possibility is to consider N=3 and k=1: (we can have several possibilities)

$$\Omega^{(k)} = k\frac{2\pi}{3}$$

$$\downarrow$$

$$k = 1 \implies \frac{2\pi}{3}$$

Example 6

(d) N=3 THIS IS JUST TO CHECK…

$$\Omega_0 = \frac{2\pi}{N} = \frac{2\pi}{3}$$

```
>> x=[10 -5];
>> fft(x,3)

ans =

   5.0000 + 0.0000i  12.5000 + 4.3301i  12.5000 - 4.3301i
```

$$X(\Omega = 2\pi/3)$$

```
>> Omega_0=2*pi/3

Omega_0 =

    2.0944

>> Omega=[0:2]*Omega_0

Omega =

         0    2.0944    4.1888
```

$$k = 1 \Longrightarrow \frac{2\pi}{3}$$

Example 6

JUST TO CHECK: $$X(\Omega) = 10 - 5e^{-j\Omega}$$

$$X(2\pi/3) = 10 - 5e^{-j\frac{2\pi}{3}} = 12.5000 + 4.3301j$$

correct !!!

Example 7

Let consider the following signal:

$$x[0] = 10, x[1] = -5, \quad x[n] = 0 \quad \text{for the rest of n}$$

Let us consider that is obtained sampling a continuos signal x(t) with sampling period T=0.1 sec.

(a) Say what is the maximum frequency of the signal x(t) that you can detect.

(b) Interpret the output of an DFT for a generic N and with N=3,4,5, as X(Omega) - FT of x[n] - and as X(omega) - FT of x(t).

Example 7

(a) If the signal x(t) has been well-sampled, we can "see" until the frequency:

$$\frac{\omega_s}{2} = \frac{2\pi}{2T} = \frac{\pi}{T} = \frac{\pi}{0.1} = 31.4159 \quad \mathrm{rad/sec}$$

Example 7

(b) **Each output of DFT from k=0,…,N-1 (fft in matlab) can be interpreted as associate to the frequencies**

$$\Omega = 0, \Omega_0, 2\Omega_0, \ldots, (N-1)\Omega_0 \qquad \Omega_0 = \frac{2\pi}{N}$$

for x[n] (discrete time), and

$$\omega = 0, \omega_0, 2\omega_0, \ldots, (N-1)\omega_0$$

for x(t) (continuos time), where

$$\omega_0 = \frac{\Omega_0}{T} = \frac{2\pi}{NT}$$

Example 7

However, recall that in continuous time we can “see” only until

$$\omega_{\mathtt{max}} = \frac{\omega_s}{2} = \frac{2\pi}{2T} = \frac{\pi}{T} = \frac{\pi}{0.1} = 31.4159 \quad \text{rad/sec}$$

therefore some outputs, in this case, have not “sense”….

$$\omega = 0, \omega_0, 2\omega_0, ..., \cancel{(N-1)}\omega_0$$

exactly only an half or half+1…. (solo mitad! o mitad+1) since fft gives you exactly N values…

$$\omega_0 = \frac{\Omega_0}{T} = \frac{2\pi}{NT} = \frac{2}{N}\omega_{\mathtt{max}} \qquad \omega_{\mathtt{max}} = \frac{N}{2}\omega_0$$

Example 7

$$X_N[k] = 10 - 5e^{-jk\frac{2\pi}{N}}$$

(b) N=3

In an exam, you have to write all the "complete" formulas with exponentials etc. here I will give you the numerical results in order to check your calculus...

$$\Omega_0 = \frac{2\pi}{N} = \frac{2\pi}{3}$$

```
>> x=[10 -5];
>> fft(x,3)

ans =

   5.0000 + 0.0000i  12.5000 + 4.3301i  12.5000 - 4.3301i
```

```
>> Omega_0=2*pi/3

Omega_0 =

    2.0944

>> Omega=[0:2]*Omega_0

Omega =

         0    2.0944    4.1888
```

and \omega_0 for continuous time? ==>

Example 7

first, a summary so far:

```
here the FFT:

ans =

   5.0000 + 0.0000i  12.5000 + 4.3301i  12.5000 - 4.3301i

Omega_0:

Omega_0 =

    2.0944

Frequencies (discrete):

Omega =

         0    2.0944    4.1888
```

Example 7

(b) N=3

$$\Omega_0 = \frac{2\pi}{N} = \frac{2\pi}{3} \qquad \omega_0 = \frac{2\pi}{NT} = 20.9440 \qquad \omega_{\mathtt{max}} = \frac{\pi}{T} = 31.4159$$

```
sampling period:

T =

    0.1000


omega_0 =

   20.9440

maximum frequency that you can see:

omega_max =

   31.4159
```

```
Frequencies (continuos):

omega =

         0   20.9440   41.8879

Just frequencies that you can see:

omega =

         0   20.9440
```

31.4159

Example 7

(b) N=4

```
here the FFT:

ans =

   5.0000 + 0.0000i  10.0000 + 5.0000i  15.0000 + 0.0000i  10.0000 - 5.0000i

Omega_0:

Omega_0 =

    1.5708

Frequencies (discrete):

Omega =

         0    1.5708    3.1416    4.7124
```

Example 7

(b) N=4

```
sampling period:

T =

    0.1000


omega_0 =

   15.7080

maximum frequency that you can see:

omega_max =

   31.4159
```

```
Frequencies (continuous):

omega =

         0   15.7080   31.4159   47.1239

Just frequencies that you can see:

omega =

         0   15.7080   31.4159
```

Example 7

(b) N=5

```
here the FFT:

ans =

  Columns 1 through 4

   5.0000 + 0.0000i   8.4549 + 4.7553i  14.0451 + 2.9389i  14.0451 - 2.9389i

  Column 5

   8.4549 - 4.7553i

Omega_0:

Omega_0 =

    1.2566

Frequencies (discrete):

Omega =

         0    1.2566    2.5133    3.7699    5.0265
```

Example 7

(b) N=5

```
sampling period:

T =

    0.1000


omega_0 =

   12.5664

maximum frequency that you can see:

omega_max =

   31.4159
```

```
Frequencies (continuous):

omega =

         0   12.5664   25.1327   37.6991   50.2655

Just frequencies that you can see:

omega =

         0   12.5664   25.1327
```

Questions?