“Periodic Brother” and Generalized Fourier Transform of a periodic signal

Discrete Time Systems

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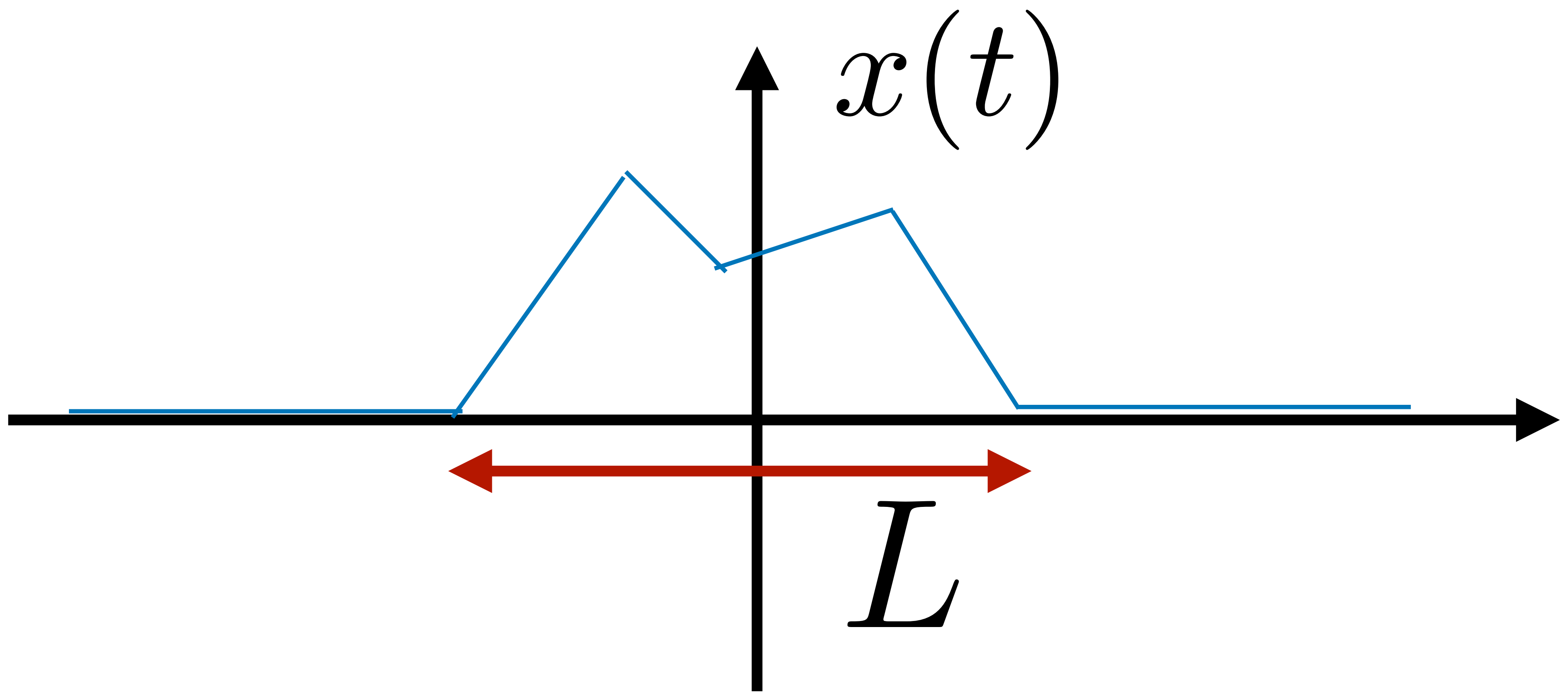
We already know …

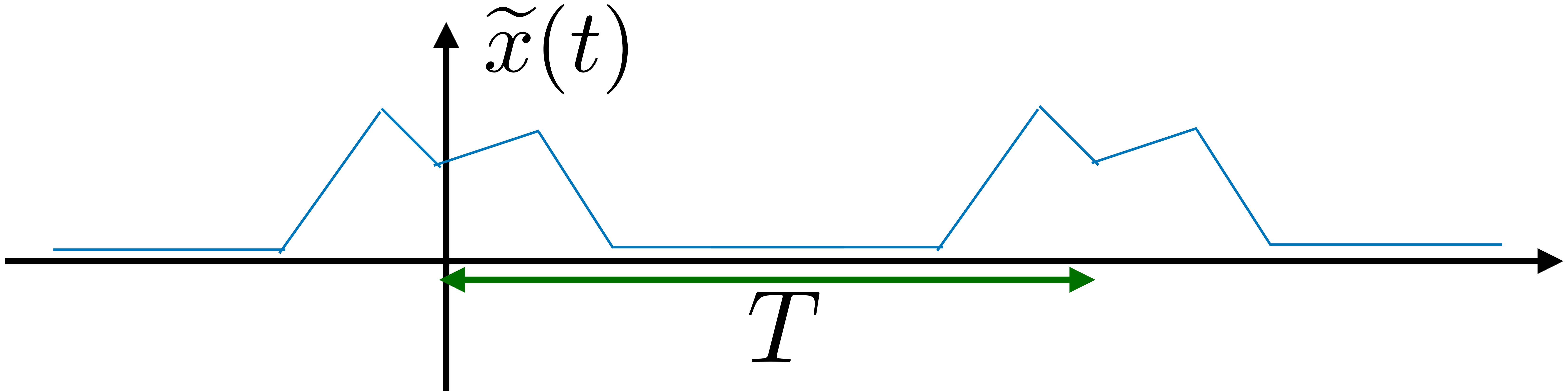
We already know the relationship between:

- the Fourier Transform of a signal ***with finite length*** x(t)
- and its "periodic brother" $\widetilde{x}(t)$

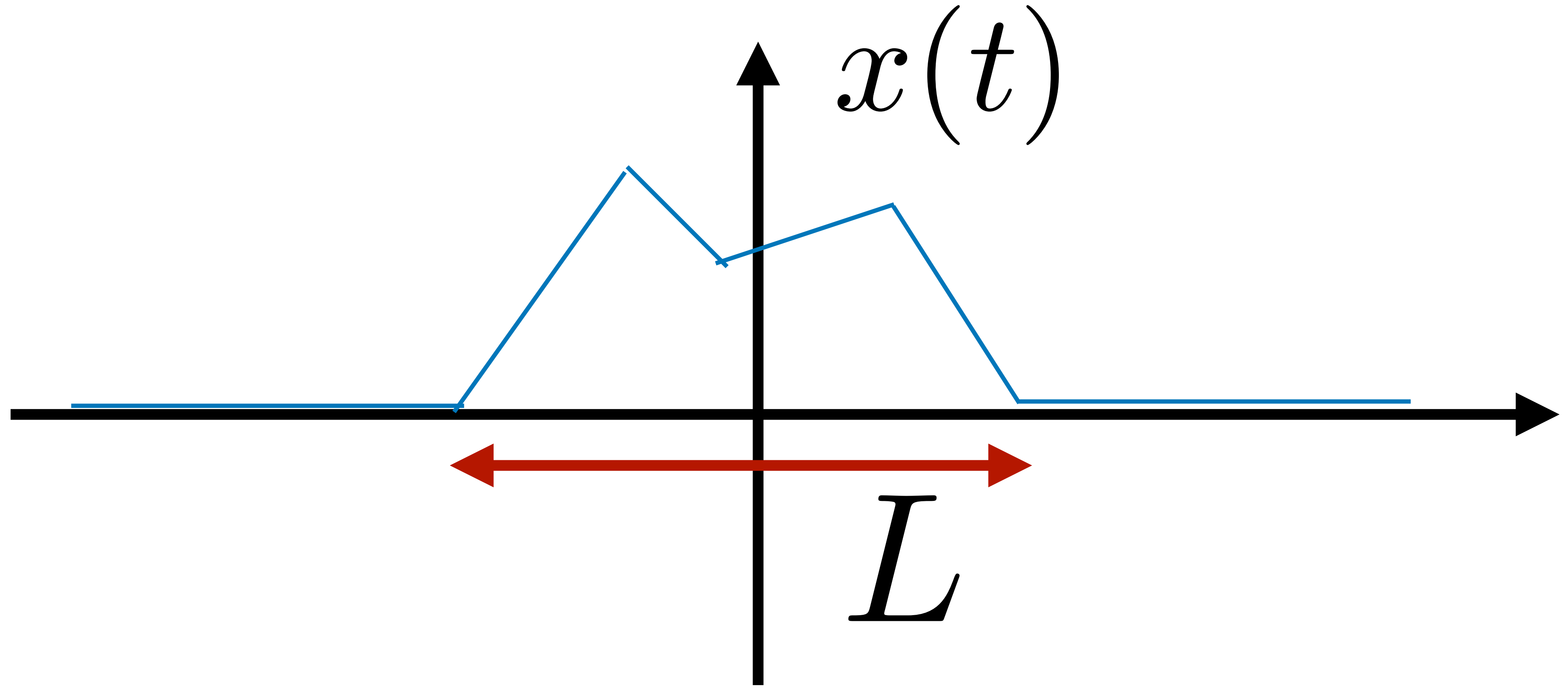


Clearly, we consider

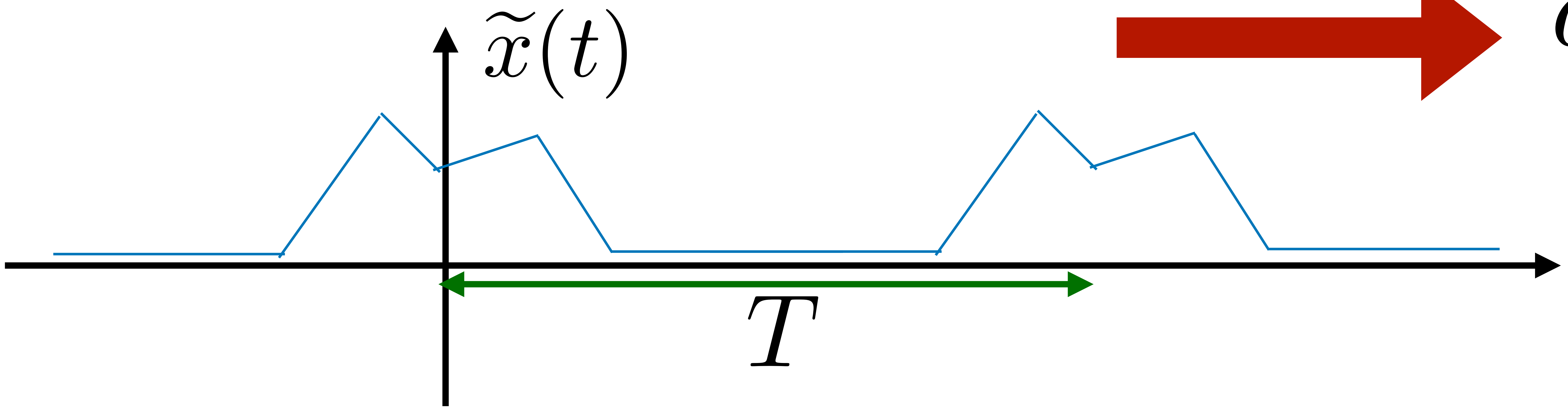
$$T \geq L$$



We already know …



$$X(\omega)$$



$$a_k = \frac{1}{T} X(k\omega_0)$$

$$\omega_0 = \frac{2\pi}{T}$$

Now, note that we can also write:

$$\widetilde{x}(t) = \sum_{k=-\infty}^{\infty} x(t - kT)$$

Moreover, we also saw that:

$$z(t) = \sum_{m=-\infty}^{+\infty} \delta(t - mT)$$

$$Z_G(\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - k\frac{2\pi}{T}\right)$$

$$\omega_0 = \frac{2\pi}{T}$$

Then, we can write:

$$\widetilde{x}(t) = \sum_{k=-\infty}^{\infty} x(t-kT) = \sum_{k=-\infty}^{\infty} x(t) * \delta(t-kT)$$

$$\widetilde{x}(t) = x(t) * \left[\sum_{k=-\infty}^{\infty} \delta(t-kT)\right] = x(t) * z(t)$$

$$\widetilde{X}_G(\omega) = X(\omega)Z_G(\omega)$$

$$\widetilde{X}_G(\omega) = X(\omega)\left[\frac{2\pi}{T}\sum_{k=-\infty}^{\infty} \delta\left(\omega - k\frac{2\pi}{T}\right)\right]$$

then we arrive again to the GFT for periodic signals!!

$$\widetilde{X}_G(\omega) = X(\omega)\left[\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta\left(\omega - k\frac{2\pi}{T}\right)\right]$$

$$\widetilde{X}_G(\omega) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty}X(\omega)\delta\left(\omega - k\frac{2\pi}{T}\right)$$

$$\widetilde{X}_G(\omega) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty}X(k\omega_0)\delta\left(\omega - k\omega_0\right)$$

$$\widetilde{X}_G(\omega) = 2\pi\sum_{k=-\infty}^{\infty}\underbrace{\frac{1}{T}X(k\omega_0)}_{a_k}\delta\left(\omega - k\omega_0\right)$$

- **Again we obtain the Generalized FT for periodic signals!**

Questions?