

Topic 2 - Part 1.1:

Other solved examples

Systems in time domain

Linear systems and circuit applications

Luca Martino — luca.martino@urjc.es — <http://www.lucamartino.altervista.org>

Based also on Professor Óscar Barquero Perez, Andrés Martínez and José Luis Rojo's slides

Example 1 of these slides

- Consider the system:

$$y(t) = x(t - 2) + x(\textcolor{blue}{2} - \textcolor{blue}{t})$$

Example 1 of these slides

$$y(t) = x(t - 2) + x(2 - t)$$

- **With memory (non-memoryless):** it depends on the past and future....

Example 1 of these slides

$$y(t) = x(t - 2) + x(2 - t)$$

- **Non-causal:** it depends on the past and future.... for instance in $t=0$:

$$y(0) = x(-2) + x(2)$$

Example 1 of these slides

$$y(t) = x(t - 2) + x(2 - t)$$

- **STABLE:** if we have a bounded input $x(t)$ the output is bounded.

Example 1 of these slides

$$y(t) = x(t - 2) + x(2 - t)$$

- For the linearity, we have first to write the output $y_z(t)$ corresponding to other input, $z(t)$, defined as linear combination of two inputs:

$$z(t) = ax_1(t) + bx_2(t)$$

Example 1 of these slides

- For the linearity, we have first to write the output $y_z(t)$ corresponding to other input, $z(t)$, defined as linear combination of two inputs:

$$z(t) = ax_1(t) + bx_2(t)$$

$$y_z(t) = z(t - 2) + z(2 - t)$$

$$y_z(t) = ax_1(t - 2) + bx_2(t - 2) + ax_1(2 - t) + bx_2(2 - t)$$

$$y_z(t) = a(x_1(t - 2) + x_1(2 - t)) + b(x_2(t - 2) + x_2(2 - t))$$

Example 1 of these slides

- We have to test if $y_z(t)$ is equal to $y_c(t)$ (combinations of outputs):

$$y_c(t) = ay_1(t) + by_2(t)$$

- where

$$y_1(t) = x_1(t - 2) + x_1(2 - t)$$

$$y_2(t) = x_2(t - 2) + x_2(2 - t)$$

Example 1 of these slides

- Replacing inside, we obtain:

$$y_c(t) = a(x_1(t-2) + x_1(2-t)) + b(x_2(t-2) + x_2(2-t))$$

- then, since:

$$y_z(t) = a(x_1(t-2) + x_1(2-t)) + b(x_2(t-2) + x_2(2-t))$$

- we have:

$$y_z(t) = y_c(t)$$

- so **it is linear!!**

Example 1 of these slides

$$y(t) = x(t - 2) + x(2 - t)$$

- For the time-invariance, we have first to compute:

$$y(t - t_0) = x((t - t_0) - 2) + x(2 - (t - t_0))$$

$$y(t - t_0) = x(t - t_0 - 2) + x(2 - t + t_0)$$

Example 1 of these slides

$$y(t) = x(t - 2) + x(2 - t)$$

- Then we to compute the output, $y_d(t)$ corresponding to a delayed input $d(t)=x(t-t_0)$:

$$d(t) = x(t - t_0)$$

$$y_d(t) = d(t - 2) + d(2 - t)$$

Example 1 of these slides

$$y_d(t) = d(t - 2) + d(2 - t)$$

- **difficult part, replacing $d(t)=x(t-t_0)$:** “replace d with x ” and ***just*** add $-t_0$ within the parenthesis....

$$y_d(t) = x(t - 2 - t_0) + x(2 - t - t_0)$$

Example 1 of these slides

- Finally, note that:

$$y_d(t) = x(t - 2 - t_0) + x(2 - t - t_0)$$

$$y(t - t_0) = x(t - t_0 - 2) + x(2 - t + t_0)$$

$$y(t - t_0) \neq y_d(t)$$

- so that, it is **time variant (not time-invariant)**

Example 2

- Consider the system:

$$y(t) = x(t) + x(\textcolor{red}{t} - \textcolor{red}{2})$$

Example 2

$$y(t) = x(t) + x(\textcolor{red}{t} - \textcolor{red}{2})$$

- **With memory (non-memoryless):** it depends on the past....

Example 2

$$y(t) = x(t) + x(\textcolor{red}{t} - \textcolor{red}{2})$$

- **Causal:** it depends on the past and present time instant

Example 2

$$y(t) = x(t) + x(\textcolor{red}{t} - \textcolor{red}{2})$$

- **STABLE:** if we have a bounded input $x(t)$ the output is bounded.

Example 2

$$y(t) = x(t) + x(\textcolor{red}{t} - \textcolor{red}{2})$$

- For the linearity, we have first to write the output $y_z(t)$ corresponding to other input, $z(t)$, defined as linear combination of two inputs:

$$z(t) = ax_1(t) + bx_2(t)$$

Example 2

- For the linearity, we have first to write the output $y_z(t)$ corresponding to other input, $z(t)$, defined as linear combination of two inputs:

$$z(t) = ax_1(t) + bx_2(t)$$

$$y_z(t) = z(t) + z(t - 2)$$

$$y_z(t) = ax_1(t) + bx_2(t) + ax_1(t - 2) + bx_2(t - 2)$$

$$y_z(t) = a(x_1(t) + x_1(t - 2)) + b(x_2(t) + x_2(t - 2))$$

Example 2

- We have to test if $y_z(t)$ is equal to $y_c(t)$ (combinations of outputs):

$$y_c(t) = ay_1(t) + by_2(t)$$

- where

$$y_1(t) = x_1(t) + x_1(t - 2)$$

$$y_2(t) = x_2(t) + x_2(t - 2)$$

Example 2

- Replacing inside, we obtain:

$$y_c(t) = a(x_1(t) + x_1(t - 2)) + b(x_2(t) + x_2(t - 2))$$

- then, since:

$$y_z(t) = a(x_1(t) + x_1(t - 2)) + b(x_2(t) + x_2(t - 2))$$

- we have:

$$y_z(t) = y_c(t)$$

- so **it is linear!!**

Example 2

$$y(t) = x(t) + x(\textcolor{red}{t} - \textcolor{red}{2})$$

- For the time-invariance, we have first to compute:

$$y(t - t_0) = x(t - t_0) + x(t - 2 - t_0)$$

Example 2

$$y(t) = x(t) + x(\textcolor{red}{t} - \textcolor{red}{2})$$

- Then we to compute the output, $y_d(t)$ corresponding to a delayed input $d(t)=x(t-t_0)$:

$$d(t) = x(t - t_0)$$

$$y_d(t) = d(t) + d(t - 2)$$

Example 2

$$y_d(t) = d(t) + d(t - 2)$$

- **difficult part, replacing $d(t)=x(t-t_0)$:** “replace d with x ” and ***just*** add $-t_0$ within the parenthesis....

$$y_d(t) = x(t - t_0) + x(t - 2 - t_0)$$

Example 2

- Finally, note that:

$$y_d(t) = x(t - t_0) + x(t - 2 - t_0)$$

$$y(t - t_0) = x(t - t_0) + x(t - 2 - t_0)$$

$$y(t - t_0) = y_d(t)$$

- so that, it is **time-invariant !!!**

Example 3

- Consider the system:

$$y(t) = \cos(3t)x(t)$$

Example 3

$$y(t) = \cos(3t)x(t)$$

- Solution:
- **without memory (memoryless),**
- **causal**
- **stable**
- **linear**
- **time variant (not time invariant)**

Example 4

$$y(t) = tx(t)$$

- Solution:
- **without memory (memoryless),**
- **causal**
- **unstable**
- **linear**
- **time variant (not time invariant)**

Questions?